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INTRODUCTION

The Problem-Solving Standard

Solving problems is not only a goal of learning mathematics but also a major means of doing so.

—National Council of Teachers of Mathematics, Principles and Standards for School Mathematics

Why Focus on Problem Solving?

Traditionally, problem solving was viewed as a distinct topic, introduced to students after they had mastered basic skills. In today’s classrooms, however, problem solving is recognized as the central focus of mathematics instruction. The ability to solve problems is the ultimate goal of mathematics. It is why we teach students to add, subtract, multiply, and divide. It is why we teach them to work with fractions, decimals, measurement, and geometry. Our goal is not for students to perform isolated computations, but rather to be able to apply their varied math skills to solve problems. But problem solving is more than just a goal of learning mathematics, it is also a critical process, woven across the entire mathematics curriculum, through which students are able to explore and understand mathematics (NCTM 2000, 52). Through problem-solving experiences, students learn to challenge their thinking about data and probability, test their ideas about number and operations, apply their skills in geometry and measurement, and evaluate their understandings of algebra. Through problem-solving tasks, students develop an understanding of math content and ultimately use that content understanding to find solutions to problems. Problem solving is both the process by which students explore mathematics and the goal of learning mathematics.
One objective of problem-solving instruction is to enable students to use their repertoire of math skills to solve problems. But it takes more than isolated math skills to be an effective problem solver. It also takes a variety of thinking skills that allow students to organize ideas, select appropriate strategies, and determine the reasonableness of solutions. It takes an understanding of how to use and adapt strategies dependent on the problem situation. And it takes an ability to reflect on how we solve problems to help us better understand our own thought processes and identify why we select and apply various strategies.

While in the past, problem solving may have been viewed as an isolated assignment (e.g., a list of word problems), problem solving today has an integrated role in the math classroom. Teachers begin lessons by posing a problem, then skills and strategies are developed throughout the lesson as the problem is explored, and those newly acquired skills allow students to successfully find a solution. Problem solving becomes both the starting point and the ending point to well-balanced mathematics lessons. Developing students’ computational skills is important, but teaching those skills in a problem-solving context ensures that students not only understand the skill but see the meaningfulness of learning the skill and understand how to apply it to real-world situations. “Problem solving is the process by which students experience the power and usefulness of mathematics in the world around them” (NCTM 1989, 75).

What Is the Problem-Solving Process Standard?

The National Council of Teachers of Mathematics (NCTM) has developed standards in order to support and guide teachers as they develop classroom lessons and create activities to build their students’ mathematical understandings. Some of those standards delineate the content to be addressed in the math classroom, while other standards address the processes by which students explore and use mathematics. Problem solving is a critical math process and the components of the NCTM Problem Solving Process Standard reflect its complex nature. Instructional programs (NCTM 2000, 52) should enable students to

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Throughout this book, we explore ways to assist students in building new math knowledge through problem-solving tasks. Highlighted problem-solving activities may be presented in math contexts as well as real-world contexts. We explore, in depth, the various problem-solving strategies that support students in finding solutions, and we identify techniques for helping students reflect and monitor their problem solving. We will dive into the NCTM process standard of problem solving in order to better understand it and find ways to bring it to life within our classrooms.
Creating Effective Problem Solvers

In my early experiences with teaching problem solving, I began much like my own teachers had, assigning problems to students and expecting them to be able to solve the problems on their own. I quickly recognized my students’ anxiety and frustration. I soon learned that assigning problems and then correcting those problems did not create successful problem solvers. I began to break down the skills needed to solve problems and find opportunities to guide my students in developing some specific strategies to help them organize their thinking. Through a combination of modeling, providing opportunities for exploration, facilitating discussions about thinking, and prompting students to reflect on their experiences, I observed the continued efficiency with which my students solved problems. The more they explored and analyzed problem-solving strategies, the more successful they became. Surprisingly, not just the most capable of my students showed progress, but all of them did. As I demonstrated various strategies to attack problems and began to let my students see math problems through visual and hands-on demonstrations, their skills improved. And my skills improved, too! The more comfortable I became at teaching problem solving, the more confident I became about my ability to help my students understand a process that had once seemed so complicated and abstract.

With an understanding of the problem-solving process and a repertoire of strategies to assist our students in dealing with problem situations, our anxiety and frustration lessen and our enthusiasm and confidence grow. Not all students can become effective problem solvers on their own, but with the help of a confident and capable teacher, all students can significantly improve their problem-solving abilities.

Developing Skills and Attitudes

Developing students’ problem-solving abilities is a challenging and complex task. It requires attention to the building of mathematical skills and thinking processes as well as attention to the development of positive attitudes toward problem solving. Both skills and attitudes must be strengthened to produce truly effective problem solvers.

Problem solving is a process, requiring students to follow a series of steps to find a solution. Although some students may intuitively follow a process, many students need to be taught how to proceed to reach a solution. Another important goal in teaching students to solve problems is assisting them in developing strategies or plans for solving problems. While choosing a mathematical operation—addition, subtraction, multiplication, or division—is frequently the way to solve a problem, alternate strategies are often needed. Helping students learn strategies such as drawing pictures, finding patterns, making tables, making lists, guessing and checking, working backward, or using logical reasoning gives students a wide variety of strategies to employ during problem solving. Problem solving requires this knowledge of strategies as well as the ability to determine when each strategy would be best used. The more our students practice these strategies, the more confident they become in their ability to solve problems and apply mathematics in meaningful ways.
The development of a positive attitude toward problem solving is crucial to student success. As teachers, we are instrumental in helping our students develop the attitudes needed to become successful problem solvers.

**Problem solving requires patience.**

It is not always possible to find a quick answer and quick answers are often incorrect. Problem solving is not judged on speed but on the reasonableness of the final solution.

**Problem solving requires persistence.**

Students may need to try several strategies before finding one that will work. Students must have confidence that they can find a solution, even if it is not immediately apparent.

**Problem solving requires risk taking.**

Students need to be willing to try their “hunches,” hoping that they may lead to a solution. Students must feel comfortable making mistakes, as problem solving is a process filled with mistakes that often lead to solutions.

**Problem solving requires cooperation.**

Students must often be willing to share ideas, build on one another’s thoughts, and work together to find a solution.

Students become successful problem solvers when they are instructed in a climate that rewards patience, persistence, risk taking, and cooperation. As teachers, we have a critical role in establishing a positive climate for problem-solving instruction.

### How This Book Will Help You

This book is designed to help you better understand the NCTM problem-solving standard. It explores problem solving as both a process through which students learn mathematics and a skill that enables them to apply the mathematics they have learned. The mathematical goals of students in grades 3–5 are specifically addressed and practical ideas for helping students become effective problem solvers are shared.

This book presents ideas for developing a problem-centered approach to teaching mathematics within your classroom. We will see how problem solving can set a context for learning math skills, can excite and engage students, and can help students discover insights and better understand math ideas. We explore ways in which problem solving enriches our math classrooms and nurtures enthusiasm, curiosity, and insight.

Within this book you will find a variety of ideas to help you better understand the problem-solving process, as well as specific strategies including Choose an Operation; Find a Pattern; Make a Table; Make an Organized List; Draw a Picture or Diagram; Guess, Check, and Revise; Use Logical Reasoning; and Work Backward. These strategies help students organize their thinking, figure out ways to approach and simplify problems, and ultimately find their way to solutions. We explore practical ways to support our students as they develop these thinking skills, knowing that the groundwork
for each strategy is laid in the primary grades, but that students in grades 3–5 refine their use of these strategies and engage in tasks requiring a more sophisticated understanding. As we investigate a variety of problem-solving strategies, we delve into their underlying skills in order to unearth the complexity and importance of each strategy. A variety of activities that are appropriate for students in grades 3–5 are shared for each strategy. Specific grade levels are not indicated on each activity, as problem-solving skills do not develop by grade level, but rather depend on students’ prior knowledge and previous exposure to each strategy. Teacher tips are shared highlighting important points to emphasize when working with students. Examples of student work are presented for each strategy, including samples of students’ communication about their problem solving. The work samples illustrate the progression of problem-solving skills, and the writing samples offer a glimpse into students’ thinking as their skills develop.

Once we have explored the problem-solving standard in depth, you will see how it connects to the math content standards in the chapter Problem Solving Across the Content Standards. Through sample classroom activities, we explore the interconnectedness of the content and process standards. We discuss sample problem-solving tasks that blend with grades 3 through 5 content in number and operations, algebra, geometry, measurement, and data and probability. Student work is shared to illustrate these lessons and you will be asked to reflect on the combined teaching of math content and the problem-solving process.

In later chapters we discuss the assessment of problem solving including the use of rubrics to assess students’ skills. We also explore real-world problem-solving tasks that challenge students to move beyond the context of math and apply their skills to everyday situations. These tasks motivate and engage students and demonstrate the meaningfulness of the mathematics they are learning. Ideas for using real-world data and materials are presented.

While this book is designed to help you better understand the NCTM Problem Solving Standard and to provide you with practical ideas and classroom activities related to the standard, it is also intended to stimulate thought about teaching and learning. Following each chapter, several questions prompt you to reflect on the content of the chapter whether alone or with a group of your colleagues. Taking a moment to reflect on the ideas presented, and relating them to your teaching experiences and your observations of your students, will help you better process the ideas and apply them to your students’ specific needs.

A very important component of this book is the inclusion of the practical resources needed to implement the ideas explored throughout the chapters. The accompanying CD is filled with a variety of teacher-ready materials to help you implement a problem-solving program in your school or classroom. Checklists, evaluation forms, scoring keys, and icons are all available as well as a variety of practice problems for your students. The practice problems range from simple to complex. Select those activities that suit your students’ level of expertise, and continue to challenge your students with more sophisticated thinking as their skills improve. And many of the activities and resources on the CD can be easily modified to suit your students’ specific needs. Change the data to make it less or more challenging, or insert familiar names and places to engage and motivate your students as they explore the problem activities.
This book was developed as a result of my readings about problem-solving theory, my reflection on current practices, and my observations on the progress of students in varied classroom settings. As a result of both research and practice, I have adapted and modified some common problem-solving techniques, developed some new activities to support problem-solving instruction, and highlighted resources and activities that are particularly effective for students in grades 3–5. It is hoped that this book will enhance your understanding of the problem-solving standard and provide you with insights and practical ideas to develop your students’ problem-solving skills. When we, as teachers, better understand the complexity and importance of problem solving, we are better able to identify, select, and design meaningful tasks for our students. It is hoped that the varied instructional practices highlighted in this book will assist you in developing your students’ skills and expanding your own understandings. Most certainly, as we reflect on and develop our teaching skills, our students’ problem-solving skills will increase as well.

Questions for Discussion

1. Were you taught how to solve math problems or just assigned problems to solve? How did you feel about math problem solving when you were a student in the math classroom? In what ways do your past experiences and attitudes about problem solving impact your teaching of problem solving?

2. If students show competence with computational skills but lack problem-solving skills, how might it affect their math achievement? What possible problems might they experience?

3. What attitudes are essential to be an effective problem solver? How might you support students in developing these attitudes?

4. What skills are essential to be an effective problem solver? How might you help your students acquire those skills?