“Many students disconnect mathematics from common sense. Transition to Algebra uses explorations, problems, and puzzles to build the logic of algebra to help students make sense of what they are learning.”

— June Mark, E. Paul Goldenberg, Mary Fries, Jane M. Kang, and Tracy Cordner, authors of the research-based Transition to Algebra series

Transition to Algebra’s 12 units include teaching guides, in-depth lessons, and assessments to support algebra students.

For more information, visit TransitiontoAlgebra.com
A New Research-Based Approach for Teaching Algebra

Help Students Build Essential Conceptual Understanding and Prepare for Success

Research shows that success in algebra:
• opens doors to more advanced mathematics
• is a predictor of eventual graduation
• acts as a gateway to a bachelor’s degree
• is linked to job readiness and higher earning once the student enters the workforce.

Designed by EDC (Education Development Center, Inc.), an international nonprofit research and development organization with extensive mathematics curriculum experience, and funded by a grant from the National Science Foundation, Transition to Algebra (TTA) uses algebraic logic puzzles and explorations to help students shift their ways of thinking from the concrete procedures of arithmetic to the abstract reasoning that success with algebra requires.

Transition to Algebra is designed to build students’ algebraic habits of mind—useful, analytic, quantitative, and logical ways of looking at the world and thinking mathematically.

Think about your algebra students. How many are highly successful? How many...
...struggle to pass and will require remediation?...pass but won’t move on to more advanced math?...pass but will continue to struggle with advanced math?

Numerous studies establish that as many as 40% of students who pass Algebra I do not return to school for Algebra II. Yet, only 40% of students who pass Algebra II go on to take advanced math courses. The remaining 60% are highly successful.

How many of your Algebra I students are highly successful?

Think about your algebra students. How many are highly successful?

To be successful with algebra, students must be able to shift their focus from the numbers themselves to reasoning about the operations on those numbers. This shift comes naturally for some students, but not for others. Students need targeted supports if they are to successfully cross the bridge from arithmetic to algebra. By giving students experiences where they can use their own logic to make sense of and succeed in mathematics, Transition to Algebra presents algebra as a commonsense, understandable subject.

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Component overview

Transition to Algebra instruction is organized around 12 curriculum units, providing support in areas where students need the most help.

Fostering Algebraic Habits of Mind see pages 2–3
Unit Overviews and Contents see pages 4–7
Correlation Between Algebra Topics and TTA’s Curriculum Units see page 7
Inside the Teaching Guides pages 8–21
Inside the Student Worktexts pages 24–29

Transition to Algebra includes

Teaching Guides
• support each unit of study
• begin with a brief but comprehensive unit overview
• offer pointers for guiding productive mathematical discussion including prompts and tips
• support TTA lessons, explorations, practice problems, and related activities pages 8–21

Answer Keys
• support each unit of study
• offer teachers easy access to worksheet solutions pages 19, 21

Series Overview
• outlines the research and guiding principles behind TTA
• describes ways of tailoring instruction to address different instructional needs page 22

Digital Resources for Teaching and Learning
• offers lesson pages and answer keys for whole-class viewing
• provides teaching tools such as templates, activities, and cutouts page 23

Student Worktexts
• include explorations, optional hands-on activities and games, and plenty of tiered practice materials
• connect to and extend a range of algebra course topics
• provide 80 combined lessons and 20 algebraic explorations pages 24–29

“Transition to Algebra is really unique in its ability to get students thinking and talking about algebra in meaningful ways.”

—Laura Dolbow, Math Teacher, Nashville, TN
Fostering Algebraic Habits of Mind

Transition to Algebra is designed to build students’ algebraic habits of mind, key mathematical ways of thinking that bring focus and coherence to students’ work with mathematics. The algebraic habits of mind central to TTA reflect the Standards for Mathematical Practice (SMP).

1. Puzzling and Persevering

Students often see mathematics as a collection of rules to know and follow. Genuine problems—in school and out—are not so cut-and-dried. Standardized tests also give problems that require students to think beyond the rules. Even ordinary word problems require students to figure out where to start and what to do next. There is no “formula” for how to do that, which is one reason students find word problems difficult. Puzzles place that particular skill—figuring out where to start—front and center. The puzzles in Transition to Algebra have been chosen strategically to support mathematical ways of thinking essential in algebra. (Aligned with SMP#4: Make sense of problems and persevere in solving them.)

2. Seeking and Using Structure

Students in beginning algebra are often taught to solve equations like $2(x + 3) + 4 = 24$ by going through a particular set of steps written in a particular way. On successive equations like $2(x + 3) + 4 = 24$ and $x + 3 = 5$, and picture the two numbers on the number line, they can “set up” that calculation mentally without needing special rules. The number line image, mentally or on paper, shows that two distances (to zero) are being added. And the sign of the result is negative, as students would expect when subtracting a larger number (53) from a smaller one (-18). The calculation $-18 + 53$ shows a different picture. (Aligned with SMP#1: Make sense of problems and persevere in solving them.)

3. Using Tools Strategically

Appropriate use of tools requires some reasoning about and picturing results before they are fully derived. In arithmetic and algebra, that means reasoning about calculations and operations and predicting how part or all of a calculation would go without carrying it out fully. For example, if students understand $-18 - 53$ as asking about the distance between $-18$ and $53$, and picture the two numbers on the number line, they can “set up” that calculation mentally without needing special rules. The number line image, mentally or on paper, shows that two distances (to zero) are being added. And the sign of the result is negative, as students would expect when subtracting a larger number (53) from a smaller one (-18). The calculation $-18 - 53$ shows a different picture. (Aligned with SMP#1: Make sense of problems and persevere in solving them.)

4. Describing Repeated Reasoning

This is the habit of looking for a pattern, exploring its mathematics, and developing a generic way (often an algebraic expression or equation) to describe it. The practice of seeking and articulating regularity is a cornerstone of algebraic thinking. Many students who can solve the algebraic equation that represents a problem can’t generate that equation. Transition to Algebra teaches students how they can generate an equation by guessing a possible solution, checking that guess, and then repeating that process for a different arbitrary guess. The goal is not for students to find an answer by guesswork, trial and error, or approximation and adjustment. The goal is for them to notice the regularity in the operations they use to check their guesses. Then students generalize the process by “calculating” the result for any value $x$, generating the equation they need to solve. (Aligned with SMP#8: Look for and express regularity in repeated reasoning.)

5. Communicating with Precision

As students develop mathematical language, they learn to use algebraic notation to express what they already know and translate words, symbols, and diagrams. Clear communication also requires the refinement of academic language as students explain their reasoning and solutions. Along with some new specifically mathematical vocabulary, this includes the use of:

- quantifiers (all, some, always, sometimes, never, any, for each, only, etc.)
- combinations and negation (not, or, and)
- conditionals (if, then, whenever, if not, etc.)

(Aligned with SMP#6: Attend to precision.)

“I am continually amazed at the understanding my kids have after using Transition to Algebra. The series helps build perseverance and has students investigate many different ways to think about problems.”
—Jennifer Outz, Math Teacher, Seminole, FL

“Transition to Algebra’s ease of use flows from its makes-sense approach. It is very well developed, coherent, and clearly presented so that teachers and students alike can see the logical progression of content.”
—Linda Ferreira, Math Coordinator, Attleboro, MA

“With Transition to Algebra I see a deeper understanding when I hear how the students talk about algebra; they talk more fluently about the math. The more advanced students are clearly forming more creative ways to solve problems. Transition to Algebra encourages different problem solving ‘angles’ of strategy, and the way students discuss their problem-solving approaches is great.”
—Lori Lustig, Math & Science Teacher, Asheville, NC

For sample lessons and additional information visit www.TransitiontoAlgebra.com
Unit Overviews and Contents

The Threading to Algebra program consists of 12 curriculum units, 80 lessons, and 22 algebraic explorations, that develop algebraic concepts and skills systematically through varied approaches and appropriate practice. Concept development and skills practice are interwoven, so that students continue to use and develop what they have learned.

Unit 1: Language of Algebra presents algebra as a language for expressing patterns and relationships. Students build intuition and develop algebraic language through “number tricks” as they move from pictures, to words, and finally to algebraic notation in order to track the transformations of an unknown starting number. Students also use mobile puzzles to learn about equivalence and to visualize the common sense behind algebraic solving steps.

Algebra topics: variables, expressions, equality, commutative property, solving equations

Note from the classroom: Unit 1 helps students embark on their algebra learning journey in a concrete fashion, and helps them build up toward the abstract.

Unit 2: Geography of the Number Line presents the number line as a tool for reasoning about integers and the relationships between integers, including order and distance. The number line is also used as a tool for making sense of the operations of addition and subtraction, first with numbers and then generalized to variables.

Algebra topics: addition, subtraction, integers, distance, variables, inequalities

Note from the classroom: The detailed use and explanation of the number line in this unit helps students build a deeper and more durable understanding of number.

Unit 3: Micro-Geography of the Number Line extends the thinking students have used in making sense of integers to decimals and fractions. Students zoom in on the number line and extend the thinking students have used in making sense of integers to decimals and fractions. Students zoom in on the number line and use mobile puzzles to learn about proportional reasoning. In the last lesson, students see how graphs can be used to depict proportional relationships.

Algebra topics: coordinate plane, functions, graphing, tables of data, linear equations, graphing relationships

Note from the classroom: Throughout the series, puzzles and explorations foster problem-solving skills. Unit 7 specifically develops student strategies to attack word problems.

Unit 4: Area and Multiplication builds a commonsense foundation for multiplying algebraic expressions by examining multiplication in the context of area. Students use area models to multiply integers and numerical expressions and then extend their understanding to multiplying algebraic expressions and identifying equivalent expressions. Students will revisit area models extensively in Unit 10: Area Model Factoring.

Algebra topics: commutative/associative/distributive properties, multiplying expressions, equivalent expressions, variables, solving equations, polynomials

Unit 5: Logic of Algebra builds facility with manipulating expressions and equations thoughtfully, logically, and accurately. Students formalize the commonsense logic they developed earlier to make sense of the rules for solving equations and systems of equations by performing only operations that preserve balance and substituting equivalent values as needed.

Algebra topics: variables, solving equations, order of operations, systems of equations, multistep equations

Note from the classroom: Unit 5 offers a variety of unique and compelling opportunities for students to successfully transition from concrete to abstract representations.

Unit 6: Geography of the Coordinate Plane builds facility with the coordinate plane, graphs, and equation graphing. Students explore coordinated data and transformations before moving on to graphing the solution points of equations. As students test points to see whether they are on the graph of an equation and hence solutions of the equation, they come to understand graphs as a collection of solution points (which supports future work with slope and distance).

Algebra topics: coordinate plane, functions, graphing, tables of data, linear equations, graphing relationships

Unit 7: Thinking Things Through Thoroughly is designed to stimulate mathematical thinking and to offer more practice solving problems, skills that take time and experience to develop. Unit 7 invites students to consider what they can determine from a context and what possibilities there are before focusing on the answer requested. Students use tables and diagrams to organize information and learn to repeat and generalize calculations to produce algebraic equations.

Algebra topic: word problems

Note from the classroom: Throughout the series, puzzles and explorations foster problem-solving skills. Unit 7 specifically develops student strategies to attack word problems.

Unit 8: Logic of Fractions builds on tools and strategies developed in earlier units, focusing on rational numbers and rational expressions. Number lines and area models help students make sense of additive and multiplicative operations with fractions, while mobile puzzles build intuition about proportional reasoning. In the last lesson, students see how graphs can be used to depict proportional relationships.

Algebra topics: decimals, fractions, multiplication of polynomials, squaring

Note from the classroom: Fractions are commonly recognized as one of the critical stumbling blocks for students in transitioning from general math to algebra. Unit 8 specifically addresses and develops mastery in these skills and concepts.

“A curriculum organized around habits of mind closes the gap between what the users and makers of mathematics do and what they say. Such a curriculum lets students in on the process of creating, inventing, conjecturing, and experimenting; it lets them experience what goes on behind the study door before new results are polished and presented. It is a curriculum that encourages false starts, calculations, experiments, and special cases… It helps students look for logical and heuristic connections between new ideas and old ones. A habits-of-mind curriculum is devoted to giving students a genuine research experience.”
Unit Overviews and Contents

Unit 9: Points, Slopes, and Lines builds on the ideas from Unit 6. Students think about two relationships between any two points: distance and slope. They find the length of the straight-line path between points, use the ratio of vertical to horizontal distance to quantify the slope of that straight-line path, and use that slope to determine collinearity, test whether points are on a line, generate new points along a line, and create an equation to describe the location of all points on a line.

Algebra topics: slope, graphing, linear equations, patterns, graphing relationships, coordinate plane, Pythagorean theorem

Unit 10: Area Model Factoring presents factoring as one kind of “un-multiplying.” Students first divide with area models, working with a given area and one dimension of the rectangle to find the other length. Students connect the models to multiplication and division equations and explore area model puzzles, thinking flexibly with the model in preparation for factoring. The factoring problems in this unit focus primarily on expressing trinomial products as two binomial factors.

Algebra topics: multiplication/division of polynomials, factoring, solving equations

Note from the classroom: Unit 10 offers a great example of how students learn to look for patterns and structures and to use those structures to make sense of the mathematics rather than looking for arbitrary rules.

Unit 11: Exponents helps students make sense of exponential growth and leads students through extending the ideas of positive exponents to negative and fractional exponents. Students see why, to keep notation consistent, it must be true that $a^1 = a$, $a^0 = 1$, and $a^{-1} = \frac{1}{a}$. Students practice using exponents in area models and in rational expressions and conclude the unit by examining rational exponents.

Algebra topics: exponents, squaring, negative/fractional exponents, square roots

Unit 12: Algebraic Habits of Mind reviews, consolidates, and extends several of the overarching themes of the year: building equations and expressions; distance and area modeling; and solving problems with logic, persistence, and strategy. Students review the use of variables and expressions, the use of the number line and area models, finding distance and slope, and the idea of solving and what a solution looks like in various contexts.

Algebra topics: review and consolidation of topics and problem-solving strategies from previous units

Correlation Between Algebra Topics and TTA’s Curriculum Units

<table>
<thead>
<tr>
<th>Pre-Algebra and Algebra 1 topics</th>
<th>Transition to Algebra units with major emphasis on topic</th>
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<tbody>
<tr>
<td>Rational Numbers and Operations</td>
<td>Unit 3: Micro-Geography of the Number Line</td>
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<td>Unit 8: Logic of Fractions</td>
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<td>Mental Mathematics</td>
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<tr>
<td>Distributive Property</td>
<td>Unit 4: Area and Multiplication</td>
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<td>Unit 10: Area Model Factoring</td>
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<td></td>
<td>Mental Mathematics</td>
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<tr>
<td>Linear Equations</td>
<td>Unit 1: Language of Algebra</td>
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<td></td>
<td>Unit 5: Logic of Algebra</td>
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<tr>
<td></td>
<td>Unit 9: Points, Slopes, and Lines</td>
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<tr>
<td>Inequalities</td>
<td>Unit 2: Geography of the Number Line</td>
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<tr>
<td></td>
<td>Unit 3: Micro-Geography of the Number Line</td>
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<tr>
<td>Ratio and Proportional Relationships</td>
<td>Unit 8: Logic of Fractions</td>
</tr>
<tr>
<td>Functions</td>
<td>Unit 1: Language of Algebra</td>
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<td></td>
<td>Unit 6: Geography of the Coordinate Plane</td>
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<td></td>
<td>Mental Mathematics</td>
</tr>
<tr>
<td>Systems of Equations</td>
<td>Unit 5: Logic of Algebra</td>
</tr>
<tr>
<td>Graphing</td>
<td>Unit 6: Geography of the Coordinate Plane</td>
</tr>
<tr>
<td></td>
<td>Unit 9: Points, Slopes, and Lines</td>
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<tr>
<td>Exponents</td>
<td>Unit 11: Exponents</td>
</tr>
<tr>
<td>Factoring Polynomials</td>
<td>Unit 10: Area Model Factoring</td>
</tr>
<tr>
<td>Quadratics</td>
<td>Unit 4: Area and Multiplication</td>
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<tr>
<td></td>
<td>Unit 10: Area Model Factoring</td>
</tr>
<tr>
<td>Word Problems and Mathematical Language</td>
<td>Unit 8: Logic of Fractions</td>
</tr>
</tbody>
</table>

This chart summarizes some key pre-algebra and Algebra 1 topics and the units that most closely relate to those topics. The structure of the TTA Units means that for any of the topics on the left, many units more than the ones listed on the right could be added because they address prerequisite skills and the habits of mind students need to succeed in algebra, and later units maintain and rehearse ideas from earlier ones.
The 12 teaching guides support daily instruction with a clear, coherent, predictable structure that helps you internalize the teaching framework. In addition to highlighting the key mathematical ideas developed across the unit, each guide includes strategies for fostering student problem solving, and suggestions for supporting classroom discussion, as well as teacher tips for addressing common problems and "what if" scenarios.

**UNIT 1**

**Language of Algebra**

In this unit, students come to understand the power of notation. Students begin this journey by using an intuitive iconic notation to understand how a number trick works—by tracking the transformations of an unknown starting number—and to make up their own tricks. Then, they replace the pictures with the words they themselves use to indicate what icons to draw. They learn that notation helps us understand and algebra makes notation easier.

Students also apply their intuitive sense of how to maintain the balance of a mobile to determine the weights of unknown objects. They then translate among the various notations while practicing arithmetic computation and learn to use the connections between simple equations and steps in the Think of a Number trick.

**Think of a Number Tricks**

In exploring number tricks, such as the one shown here, students encounter many of the key ideas of introductory algebra. They express practical numbers with generic notation (the buckets) to see how the trick works for any number, and in describing the pictures accompanying multiplications or divisions, they demonstrate the natural logic of the distributive property that they will later encode and learn to apply formally.

Imagine playing the following trick with your students, posing each step to allow for mental calculation:

- Think of a number:
  - Add 3.
  - Double what you got.
  - Subtract 4 from that.
  - Divide your result by 2.
  - And subtract the number you first thought of.

How could you know the result is 1? You know there is a mathematical reason this trick works, and you may have even used some notation to figure it out. In this unit, students will use tables with various notations to help them understand this number trick and to even learn to make up their own. Students will also see Think of a Number trick notation as a try to determine a partner’s starting number after operations have been performed on it (e.g. multiply by two, then add three). While students initially use pictures of buckets to track and add the steps, as with any other pictures as variables and “concrete” algebraic notation as a natural way to abbreviate these pictures. They then translate among the various notations while practicing arithmetic computation and learn to see the connections between simple equations and steps in the Think of a Number trick.

If Ben got 13 when the picture was

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<table>
<thead>
<tr>
<th>Instructions</th>
<th>Pictures</th>
<th>Algebraic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Think of a number</td>
<td>![Picture of a bucket]</td>
<td>$x$</td>
</tr>
<tr>
<td>Add 3</td>
<td>$x + 3$</td>
<td>$x + 3$</td>
</tr>
<tr>
<td>Double what you got</td>
<td>$2(x + 3)$</td>
<td>$2x + 6$</td>
</tr>
<tr>
<td>Subtract 4</td>
<td>$(2x + 6) - 4$</td>
<td>$2x + 2$</td>
</tr>
<tr>
<td>Divide result by 2</td>
<td>$(2x + 2)/2$</td>
<td>$x + 1$</td>
</tr>
<tr>
<td>Subtract the number you first thought of</td>
<td>$(x + 1) - x$</td>
<td>1</td>
</tr>
</tbody>
</table>

**Mobile Puzzles**

Mobile puzzles, with their real world.

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For sample lessons and additional information visit www.TransitiontoAlgebra.com
For example, using a for circle, b for star, and c for diamond, here are some of the equations suggested by this mobile:

- 4a = 24 (because half of the weight of the mobile is on the left side)
- 4b = 16
- 2c = 12 (because half of the weight of the right two strings is 12)

Mental Mathematics

This unit’s mental mathematics focuses on doubling and halving. A major purpose is to build, from intuitive use, a strong “put same” of the distributive property, which students also encounter as they manipulate sets of icons in their number tricks. This property will be generalized and expressed formally in later units. This doubling and halving in the first unit offers arithmetic that is especially tractable and useful, and students will use it constantly with the mobile puzzles.

Lesson 1 begins with a Think of a Number trick instead of Mental Mathematics, but after the first day, start each day’s session (whether or not you are starting a new lesson) with five minutes of mental mathematics.

Exploration

In the Color Towers Exploration, students explore a combinatorics problem with a focus on being systematic in their approach and communicating their process of counting by organizing possibilities. The Exploration may be used at any time in the unit, but it would work best between Lessons 4 and 5.

Related Activity

Because the logic of mobile puzzles mirrors the logic of equations, deeper work with these puzzles supports deeper understanding of the logic of solving equations. Allowing students to create their own mobile puzzles offers a different perspective on what it means for a mobile to balance and how changes to a mobile affect the balance.

Assessments

The Snapshots Check-in may be used after either Lessons 3 or Lesson 4. Use the Unit Assessment after completing all of the lessons, working through some of the Unit Additional Practice problems, and using the Student Reflection Questions on page T19.

Mental Mathematics • Activity 3

Halving 2- and 3-digit numbers

PURPOSE

Students increase their ability to find multiple numbers related to a common number. The activity builds on their earlier work with doubling and the distributive property, and extends to 3-digit numbers (e.g. half of 642 is the sum of the halves of 600, 40, and 2).

Introduce

“Again, we’re halving—finding half of a number. For example, if I say 600, you’d say 300. Ready?”

About the sequence

Even numbers between 10 and 20 are “easy” facts. Halving them, with or without an even hundred digit, is introduced in Step 1. Step 1 requires just two calculations (e.g. if halve 618, students must find half of both 600 and 18, and say those results). Step 2 requires students to juggle those values for halving (e.g. for 602, students must halve 600, 40, and 4).

Step 1: Include one number less than 20. That is the same number with an even hundred place.

<table>
<thead>
<tr>
<th>Number</th>
<th>Half</th>
<th>Double</th>
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</thead>
<tbody>
<tr>
<td>12</td>
<td>6</td>
<td>12</td>
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<tr>
<td>16</td>
<td>8</td>
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<td>602</td>
<td>301</td>
<td>1204</td>
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</table>

Step 2: Include five-digit numbers in which each digit is even.

<table>
<thead>
<tr>
<th>Five-Digit Number</th>
<th>Half 5-Digit Number</th>
<th>Double 5-Digit Number</th>
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<tbody>
<tr>
<td>200 100 800</td>
<td>100 500 400</td>
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Mental Mathematics • Activity 3

T35

Every Transition to Algebra lesson begins with a brief (3- to 5-minutes), lively, and highly focused mental mathematics activity. Drawing attention to algebraic patterns while supporting students’ arithmetic fluency, these activities are typically verbal exercises with the whole class, though sometimes they take other formats such as choosing one row of students at a time.

Tips include useful pointers for running a successful mental mathematics activity or provide other support.

For sample lessons and additional information visit www.TransitiontoAlgebra.com
Lesson Support

Lesson at a glance offers a brief overview of the lesson, including a list of the materials needed and pacing suggestions.

The context-setting purpose statement explains the lesson’s mathematical principles and how the lesson addresses the unit’s learning goals.

Each lesson begins with mental mathematics. See the sample on page 11 of this brochure.

The lesson launch sets the tone for the class through a teacher-led demonstration or student collaboration.

Algebraic habits of mind boxes address how these mathematical ways of thinking relate to lessons and share ideas for encouraging these habits.

Lesson 3: Balancing Mobile Puzzles

Purpose

Lesson 3 introduces mobile puzzles as a fun and intuitive context for reasoning about solving equations using manipulatives, substitutions, division, and multiplication. In unit 1, lessons and in unit 1: Algebra, students will explore the solving process in more detail and make direct connections between the logic of balancing mobiles balanced and the steps used in solving an algebraic equation.

Launch: Thinking Out Loud and Discussion

Write the equation $\begin{array}{c} \text{bucket} \\ + \text{weight} \end{array} = \begin{array}{c} \text{weight} \\ + \text{hand} \end{array}$ on the board, and ask three students to read through the first dialogue. If helpful, have a different set of students re-read the dialogue. Often these dialogues present new information or a different way of thinking, and it may help students to hear the information more than once.

Pose PROBLEM 1 for discussion. Elicit responses from several students prompting them to explain their answer.

Thinking Out Loud

Ask students to describe the problem to a partner. Ask students to explain the bucket is 3 and the hand is 1.

Encourage students to explain how they came to their conclusions, and how they can use their conclusions to solve the equation.

Discuss & Write What You Think

Compare and discuss the different solution methods. Ask students to write their own explanation in their copy of the Student Worktext.

Support and Using Structure

Offer students a chance to work in pairs or groups to solve the equation. Ask students to explain their solution methods to a partner.

Algebraic Habits of Mind

Students may feel satisfied with the explanation, “because the bucket is 3.” Remind them of the problems from Lessons 2 that had multiple solutions, and have them explain how they checked their answers or used intermediate steps like $2 \times 3 = 6$ to justify their reasoning. Encourage each student to write their own explanation in their copy of the Student Worktext.

Have students study the second dialogue together (perhaps twice), and invite the class to discuss the similarities and differences between the two solution methods. Draw attention to the structure of the mobile, the correspondence between the mobile and the equation, and the reasoning behind the solution methods presented.

Student Problem Solving and Discussion

Some mobile puzzles can be solved using multiple approaches. Students may choose to start with the shape with known values and deduce the values of other shapes. Or, they may use a mix-up, heightening the total weight of the (or beam) to determine the weight of each string below. Whatever the method, it helps students to use sound logic and, as much as possible, communicate their reasoning to others.

The handwriting script in PROBLEM 2 is there to help students develop effective mobile-solving strategies. Students don’t necessarily need to do this themselves, but if they have trouble tracking their progress, encourage them to use this notation to keep track of what they know. Model this notation for students as you solve mobiles as a class.

As a final minireview at the end of the class, have the class discuss some of the more difficult mobiles such as PROBLEM 5-6. Consider drawing attention to the use of the bucket or the mobile in problem 11 and how its weight affects the puzzle.

As students consider a mobile, use these questions as appropriate to facilitate their solving process:

- What does the variable mobile weight equal?
- What does each side of the mobile weigh?
- What weights do you already know?
- Can you predict any other weights using this information?

Related Activity: Making Mobiles

In the last minute of each remaining class session in this unit, allow students to explore the process of creating a mobile puzzle and to draw and solve their mobile puzzles using the related activity on page 25 of the Student Worktext. In this lesson, the activity may be done either before or after the Snapshot Check-In.

For sample lessons and additional information visit www.TransitiontoAlgebra.com

These teacher tips suggest how to structure the lesson and explain the mathematical content.

What If . . .

What if students don’t want to read in front of the class? Allow students time to practice their lines before performing. You may also permit some students to read their lines aloud at first, though a standard performance is generally more engaging to other. Ask if the dialogue was more fun or engaging. Consider the dialogue about push as a fun way to engage students in the content. 

What if students don’t understand the dialogue?

Ask students to describe why they think it’s hard to understand the dialogue. If helpful, have the class discuss the problem of the dialogue. You may also permit some students to read their lines aloud at first, though a standard performance is generally more engaging to other. Ask if the dialogue was more fun or engaging. Consider the dialogue about push as a fun way to engage students in the content.

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**Explorations**

**Exploration at a Glance**
Each group will need six blocks: five white blocks and one blue block. Students may see the solution fairly quickly and may be able to explain that there are six different towers. Because there are six possible positions for the one blue block. If students don’t offer the solution, ask: “How do we know if we have found all different towers?” Let the answer be creative and make their own discoveries.

**Mental Mathematics** Begin each day with the next Mental Mathematics activity in the sequence (pages T32–T40). Even on Exploration days, continue to work through the trajectory of activities on doubling and halving.

**Launch: Introduction to the Activity**
Hold up six blocks: five white blocks and one blue block (or whatever two colors you have). Ask: “How many different six-story towers can we make out of these blocks?”

Students may see the solution fairly quickly and may be able to explain that there are six different towers because there are six possible positions for the one blue block. If students don’t offer the solution, ask: “How do we know if we have found all different towers?” Listen to several responses, but refrain from evaluating them yet. Allow students to draw their own conclusions, and for now, just let them voice their ideas. Encourage attempts at communicating and try to have the students themselves debate the ideas. Tell students that this kind of idea-generating conversation is a model for how you would like the students to be communicating with each other during the Exploration. In the Student Workbook, discuss the example with Four Blocks (two blue and two white) at the top of the page. This demonstrates that students can color in the spaces to show the different towers.

**Student Exploration and Discussion**
Give each group of blocks (blue and four whites and have students start the Exploration. Invite students to draw how they organized their towers to find the total number of possible towers. Request explorations from multiple students, even if they share very similar findings. As students hear and produce more mathematical language, acknowledge ways that students are refining their language or responding to and building on the observations of others.

**What If . . .**
What if students give up quickly? What if students start solving the problem but are, even after, too focused on the patterns they notice? What if students think they have solved the problem but realize they haven’t yet? Encourage students to share their ideas to build on what they’ve done so far. Minimize the need to “figure out what it is.”

**Possible Approach #1:**
Fix one dark block in the first space and then cycle the second dark block through all possible positions. Then move the fixed block in the second space and cycle the second through all possible positions below it. Repeat this process, leaving the fixed block by one each time, until there are no lower spaces in which to move the fixed block while still having space for the second block.

**Possible Approach #2:**
Placethe two dark blocks in the first two spaces and cycle them through the spaces below us a group of two. Then place them in the first and third spaces, as they are spaced one apart, and cycle them through the spaces below while maintaining their relative position. Repeat this process, increasing the space between them by one each time.

**Two Possible Organizational Methods**

**Color Towers**

**PURPOSE**
Student study a combinatorial problem with a focus on using systems in their approach and communicating their process of counting by organizing the possibilities.

**Mental Mathematics** Begin each day with the next Mental Mathematics activity in the sequence (pages T32–T40). Even on Exploration days, continue to work through the trajectory of activities on doubling and halving.

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**Mental Mathematics** Begin each day with the next Mental Mathematics activity in the sequence (pages T32–T40). Even on Exploration days, continue to work through the trajectory of activities on doubling and halving.
Students explore more than they are usually encouraged to. Having kids move through an activity and be allowed to fail was unique and affirming. It was different from how you’d teach a traditional program. Having the students learn and take ownership of their learning was important.”

—Katie Conugh, 9th Grade Math Teacher, Lowell, MA
Snapshot check-ins and their corresponding answer keys are provided in their respective teaching guides.

In addition to assessment answer keys provided in the teaching guides, comprehensive **answer key** booklets are provided with all of the solutions for each student worktext.

**Formative Assessment**

**Student Reflections & Snapshot Check-in**

Snapshot check-ins offer teachers and students an opportunity to gauge what students know, discuss any difficulties as a class or individually, and learn from those discussions before moving on.

Through reflection activities and student discussions, the snapshot check-ins offer timely feedback on what students have learned.

**Sometime during Lesson 3 or 4, taking care not to disrupt a successful student problem solving session, ask students to reflect on their learning:**

**What are some things you’ve learned so far in this unit?**

**What questions do you still have?**

Assess student understanding of the ideas presented so far in the unit with the **Snapshot Check-in** on page T26. Use student performance on this assessment to guide students to select targeted Additional Practice problems from this or prior lessons as necessary.

**So far in Unit 1, students have:**

- Solved shape equations.
- Reasoned with partial information about **Think of a Number** tricks.
- Worked backward from **Think of a Number** trick results to determine the starting number.
- Learned to solve mobile puzzles using logic to maintain balance.

**Students have also focused on the following Algebraic Habits of Mind:**

- **Puzzling and Persevering**—Students have solved mobile puzzles without an algorithm for solving and seen that, in some cases, there can be more than one pathway to the solution.
- **Seeing and Using Structure**—Students have attended to the structure of mobiles as they solved them, noting relationships between shapes and considering how mobiles can be translated into equations.
- **Communicating with Precision**—Students have practiced communication through class discussions, **Discuss & Write What You Think** questions, and **Thinking Out Loud** dialogues that model effective mathematical communication.

**Instructions**

**Pictures**

**Jing**

Only 12 3

**Hali**

Only 28 6 28

**Jacob**

Only 28 6 28

Think of a number.*

Add 3.

9

3

11

DOUBLE**

Multiply by 2.

12

3

14

Divide by 2.

5

5

6

Subtract your original number.

6

9

6

If Asher got 9 when the picture was , what was his original number ( )?

6

If Ben got 32 when the picture was , what was his original number ( )?

8

If + 6 = 18, then = ______.

12

If 6 = 27, then = ______.

9

Figure out what the trickster said and what they were picturing, and find the missing results and starting numbers.

**Think of a number.**

Multiply by 3.

12

21

36

24

35

0

9

5

3


**Professional Support: Inside the Teaching Guides**

**Snapshot check-ins** offer teachers and students an opportunity to gauge what students know, discuss any difficulties as a class or individually, and learn from those discussions before moving on.

**Through reflection activities and student discussions, the snapshot check-ins offer timely feedback on what students have learned.**

**For sample lessons and additional information visit** www.TransitiontoAlgebra.com.
Throughout Unit 1, students have focused on the following Algebraic Habits of Mind:

- **Represented algebraic expressions with pictures.**
- **Translated mobiles and shape equations into algebraic expressions.**
- **Used expressions together with numeric results from Think of a Number tricks.**
- **Created expressions to describe results in Think of a Number tricks.**

Before the Unit Assessment, ask students to reflect on the following:

- What are some things you learned in this unit?
- What questions do you still have?
- Reflections can be done orally, on paper, or some combination of both.

Throughout Unit 1, students have focused on the unit’s learning goals in a summative fashion and provide information about student mastery. The assessments are generally two pages in length, and cover content from the important Stuff sections of the student worktext.

**Unit assessments** evaluate students’ success with the unit’s learning goals in a summative fashion and provide information about student mastery. The assessments are generally two pages in length, and cover content from the important Stuff sections of the student worktext.

**Unit assessments** and their corresponding answer keys are provided in their respective teaching guides.

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"With Transition to Algebra my students were able to have a deeper understanding of algebraic concepts and were able to apply them directly to Algebra 1. They developed a significant level of confidence, raising hands, participating in their Algebra 1 class."

— Kelley Donoghue, 9th Grade Math Teacher, Lowell, MA

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"I am very excited about the kids’ ability to write and talk about the math, which specifically will drive the kids’ test score improvement when they hit the PARCC test."

— Gail DeBusk, Math Teacher, Kingston Springs, TN
A Habit of Mind Approach

Rather than building natural skills, pursuing particular topics or teaching a collection of stand-alone techniques, Transition to Algebra is designed to expand, modulate, and develop a set of algebraic reasoning skills that are essential to problem solving. Using only the language of algebra, students see the "something" (the expression) with a new set of eyes. As a result, they understand the calculation as asking for the total distance and sign, they can use only the language of algebra, and they understand the calculation by using only the language of algebra.

Transition to Algebra focuses on the algebraic habits of mind that are components of five of the Standards for Mathematical Practice (SMP) listed in the Common Core State Standards (CCSS).

Digital Resources for Teaching and Learning

A wealth of unit-specific digital resources support your teaching throughout the year. This rich assortment of teaching tools includes game and activity materials, projection pages, and assessments. Offering daily support, these resources will help you establish a structured learning environment that fosters independence and self-direction.

Series Overview

The Series Overview explains the guiding principles behind Transition to Algebra’s habits of mind approach and strategies for cultivating a collaborative, problem-solving classroom culture. It also includes a detailed description of the series’ components and instructional design as well as insider tips gleaned from four years of classroom development.

For additional professional development options visit www.TransitiontoAlgebra.com

Transition to Algebra is designed to build students’ algebraic habits of mind, key mathematical ways of thinking aligned with the Standards for Mathematical Practice.

Implementation Options

As you plan for using Transition to Algebra in your classroom, you may want to think about the structure in which you will be using the material. Your students may need algebraic support and professional development and support for teacher implementation. In this section, we provide additional information to support you in this planning including different options for using the materials.

Transition to Algebra is a full-year course that was designed specifically to respond to a need for a series of resources, including multiple algebraic habits of mind that develop over the year. The course was designed and tested in settings where teachers used the course along with other resources available, providing a variety of options for using the materials. Here are three recommendations that vary depending on the purpose of the course:

If your course is one semester, selection of units should depend on the purpose of the course:

- Taught concurrent with Algebra 1, use of all 12 TTA units over
- A Habits of Mind Approach

The teaching resources at the end of each teaching guide and student workbook are also provided in an electronic format that is easy to search, review, and print.
Dear Student,

Algebra is two things: a convenient language for expressing patterns and relationships you know and a logical system for figuring out things you don’t yet know.

This book is not about “rules” and “facts” but shows you how you can make sense of algebra without rules that seem random. Mathematics is always perfectly logical. This book is designed so that you see how it all makes sense.

In Unit 1, you’ll learn to describe mathematical relationships and patterns using pictures, words, and algebraic language. You’ll use your own common sense and logic to work out the solutions.

Puzzles will be an important part of your work in this course because they don’t tell you where to start. Like life, they require you to explore, be clever, and stick with it. The goal is for you to learn to use the logical thinking you already have to solve the new kinds of problems you are learning about in algebra. Enjoy!

—The Authors

Thinking Out Loud

Michael, Lena, and Jay are working on this problem.

Thinking Out Loud

If \( \begin{array}{c}
\star = 10 \\
\square = 20
\end{array} \), then what number(s) can the \( \square \) represent?

Lena: I never thought about it before, but that statement uses an equals sign even though the two sides don’t look the same!

Jay: They aren’t the same, but they do have the same value. That’s what it means to be equal.

Michael: So the question is, when do they have the same value? When will it be true?

Lena: We need to figure out what has to be in a bucket for the two sides to be the same. Since each bucket holds the same amount, I can remove the matching buckets because that won’t affect the balance.

Michael: Right, so let’s remove the matching ones. (Lena removes \( \star \) and \( \square \), and Jay writes the new equation \( \begin{array}{c}
\star = 15 \\
\square = 8
\end{array} \).)

Jay: That leaves us with \( \begin{array}{c}
\square = 8 \\
\square = 5
\end{array} \), so they’re equal!

Discuss & Write What You Think

If \( \square = 2 \), would the statement “\( \begin{array}{c}
\star = 40 \\
\square = 8
\end{array} \)” be true? Why or why not?

Thinking Out Loud

Jay: Yeah! We can imagine the buckets and ones hanging from the strings. Just like before, the bucket holds my original number, but we can’t see inside. Anyway, I saw that the top of each side is a bucket and the bottom has the same as the 3 ones on the left. That’s how the chunks of stuff match up!

Lena: We need to figure out what has to be in a bucket for the two sides to be the same. Since each bucket holds the same amount, I can remove the matching buckets because that won’t affect the balance.

Michael: Right, so let’s remove the matching ones. (Lena removes \( \star \) and \( \square \), and Jay writes the new equation \( \begin{array}{c}
\star = 5 \\
\square = 10
\end{array} \).)

Jay: That leaves us with \( \begin{array}{c}
\square = 10 \\
\square = 5
\end{array} \), and we remove the matching ones. (Lena draws arrows.)

Michael: So \( \begin{array}{c}
\square = 3 \\
\square = 2
\end{array} \). That makes sense! If \( \square = 3 \), then \( \begin{array}{c}
\star = 9 \\
\square = 5
\end{array} \) and \( \begin{array}{c}
\square = 5 \\
\square = 5
\end{array} \). So they’re equal!
For sample lessons and additional information visit www.TransitiontoAlgebra.com

Inside the Student Worktexts

These algebraic habits of mind boxes highlight for students when particular habits of mind are useful—either when they are demonstrated by a student in a thinking-out-loud dialogue or when students would benefit from attending to one of the habits of mind while solving problems in the text.

By regularly requiring students to connect pictures, verbal descriptions, graphs and equations, TTA directly supports SMP2: Reason abstractly and quantitatively.

Important Stuff (the green tinted box) contains the lesson’s core ideas. Subsequent lessons and unit assessments rely on the learning in this section.

Transition to Algebra classrooms are lively and active places. They involve students working individually and in small groups solving problems, sharing ideas, and honing their mathematical thinking. Puzzles and problems help foster a mathematical classroom culture, particularly for students making the transition from arithmetic to algebra.

Lesson 3: Balancing Mobile Puzzles

Algebraic Habits of Mind: Seeking and Using Structure

Jay looks at the mobile and sees that a whole chunk of the left side (the first three marbles) matches with one piece on the right side. Switching between looking at individual objects and groups of objects can make problems simpler to solve.

Every beam in these mobiles is balanced. The strings and the beams weigh nothing. Find the weight of each shape.

1. $a = 1$
2. $b = 2$
3. $c = 3$
4. $d = 4$
5. $e = 5$
6. $f = 6$
7. $g = 7$
8. $h = 8$
9. $i = 9$

Total weight of mobile

$12 + 24 + 24 + 36 = 96$

STUFF TO MAKE YOU THINK

If $a + b + c = 10$, what is $d$?
If $a = 3$, does $d + 8 = a + x + 2$?
If $b + 6$, does $h + 7 = b + 8 + 1$?
If $y + y + 1 = 15$, what is $y$?

TOUGH STUFF

Find the total weight

Stuff to make you think problems extend the learning in the lessons, highlight surprising connections and satisfying patterns, and support differentiated instruction.

The numbers, equations, graphs, and other modeling tools used throughout TTA directly support, SMP4: Model with mathematics and, SMP5: Use appropriate tools strategically.

Tough stuff problems go farther with the material from the other two sections, use more challenging numbers, or have less straightforward solutions.

“Student math phobia seems to be overcome with Transition to Algebra’s puzzles as an entry point. The sequence of content and skills build up at a very nice and thoughtful pace.”
—Kate Clapp, Math Teacher, West Hartford, CT

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Inside the Student Worktexts

**Exploration: Color Towers**

For example, if you have four blocks, there are exactly six different towers you can build with two blue blocks and two white blocks.

You have six blocks. Two are blue and four are white. How many different ways can you arrange the six blocks?

Organize your solutions and describe how your organization method helps to show that you have found all possible solutions.

This space is for experimenting. Keep track of solutions you find, and look for similarities or patterns that will help you organize your solutions. You may not need all the towers. How can you be sure you're done?

**Algebraic Habits of Mind: Communicating with Precision**

Describing your process out loud to a friend can help you make sense of your own work.

Each exploration starts with a pattern or problem. Many of these patterns are modeled using manipulatives that encourage thinking that is not always as easily achieved by drawing or writing.

There are also additional practice problems at the end of the unit that review core concepts and help students prepare for the unit assessment.

These boxes call students' attention to the algebraic habits of mind they are using in exploring the problem.

Additional practice problems and puzzles are included with each lesson for use at any time in the unit. These materials are versatile and can be used to address individual learning needs, assigned as homework, or reviewed in preparation for the unit assessment.

"With Transition to Algebra, student engagement was off the charts. Students who hated math really got engaged.…Even with students who didn’t speak English, the visual nature of Transition to Algebra allowed them to get the algebraic concepts." — Maryann Finn, Math Coordinator, Malden, MA
Developed by Education Development Center (EDC), Transition to Algebra is a classroom resource that approaches algebra instruction differently. Instead of reteaching the same algebra curriculum in the same way, Transition to Algebra uses logic puzzles, problems, and explorations to help teachers uniquely build students’ mathematical ways of thinking. It invites students to experience the coherence and meaning of mathematics—perhaps for the first time.

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