Now I Get It

Strategies for Building Confident and Competent Mathematicians, K–6

SUSAN O’CONNELL

HEINEMANN
Portsmouth, NH
## Contents

Acknowledgments ix  
Introduction xi

### I. BALANCING THE MATH PROGRAM—CONCEPTS, SKILLS, AND APPLICATIONS

- ONE Problem Solving as the Focus of Math Instruction 1  
- TWO Helping Students Understand Basic Facts and Computations 15  
- THREE Developing Math Concepts Through Manipulatives 25  
- FOUR Using Children’s Literature to Teach Mathematics in Context 33

### II. REFINING UNDERSTANDING THROUGH COMMUNICATION

- FIVE Guiding Understanding Through Teacher Talk 41  
- SIX Developing the Language of Math 51  
- SEVEN Students Talking to Students—The Role of Cooperative Learning 61  
- EIGHT Writing About Mathematical Understanding 69

### III. SUPPORTING AND ENHANCING INSTRUCTION

- NINE Reaching Out to All Students Through Differentiated Instruction 83  
- TEN The Role of Technology in Enhancing Teaching and Learning 93
## CONTENTS

<table>
<thead>
<tr>
<th>ELEVEN</th>
<th>Maximizing Parent Involvement</th>
<th>105</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Conclusion</td>
<td>113</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>115</td>
</tr>
</tbody>
</table>

### ON THE CD-ROM

Appendices—Practical Classroom Resources

- A  Sample Strategy Problems
- B  Activities to Review Basic Math Facts and Computation Skills
- C  Manipulative Templates
- D  Children’s Literature Related to Math Concepts
- E  Math Vocabulary Lists
- F  Word Boxes
- G  Bingo Cards
- H  Math Writing Tasks
- I  Rubrics
- J  Graphic Organizers
- K  Math Centers
- L  “Must-See” Math Websites
Traditionally, learning computations was a matter of memorizing basic facts and computational procedures. Teachers demonstrated algorithms and then students practiced until they were able to repeat the procedures. While students who possessed strong memory skills and an ability to bring their own understanding to computations fared well with those techniques, many others were unable to master mathematics simply by memorizing a series of rote skills and procedures.

Students must know basic facts and how to compute fluently, but they should also understand basic operations and computations. The National Council of Teachers of Mathematics (NCTM) affirms that “understanding numbers and operations, developing number sense, and gaining fluency in arithmetic computation form the core of mathematics education for the elementary grades” (2000, 32). This chapter explores instructional strategies that support students’ understanding and retention of computational skills.

Why Is It So Important to Understand Basic Facts and Algorithms?

Each skill or concept students learn builds on their prior knowledge. At the core of students’ math understandings are basic computational understandings. Without knowledge of basic computations, how can students explore concepts related to fractions or decimals or apply ideas to money, measurement, or geometry? At the core of understanding mathematics are the basics of numbers and operations. A strong foundation allows students to connect new knowledge and build more complex mathematical understandings.

While knowing facts and procedures is an important skill, it is the ability to apply knowledge that is most useful to students. It is the ability to use facts to solve problems that is the optimal goal; recall alone is only for classroom exercises. To apply and retain computational skills, there must be a high level of understanding of them (Fennema and
Romberg 1999). Before students practice and memorize, they should understand the ideas behind procedures. “Developing fluency requires a balance and connection between conceptual understanding and computational proficiency” (NCTM 2000, 35).

Students who merely memorize facts and procedures without understanding may quickly forget them; they will have no way to find an answer if facts and procedures are forgotten. Corwin contends that “children may learn arithmetic procedures by repetition alone; if so, their only tool for recalling how to find solutions is their memory. Mathematics, rather than resting on a rich base of exploration, discovery, conversation, and common sense, may rest exclusively on the relatively weak platform of memory” (1996, 2). Students who engage in conceptual discussions and investigations in which they explore and invent meaningful computational strategies are building a strong foundation for necessary future mathematical skills.

**What Are Invented Procedures and How Are They Helpful?**

In the past, teachers simply showed students how to do traditional algorithms; however, in today’s math classrooms, we see evidence of students building strong understandings of numbers and operations through activities that allow them to invent procedures that make sense to them. Rather than memorizing a series of rules, procedures, or formulas, students make sense of the math in the problem and figure out ways to find solutions. Discussions about students’ methods allow them to share their thinking, refine their ideas, and test their procedures to see if they work in all situations. Through these investigations and discussions, students invent algorithms that make sense to them. A strong computational foundation can result from activities in which students learn computations through invented procedures and classroom discussions, as well as standard algorithms (NCTM 2000).

A group of second graders explored a problem about seating arrangements for a pizza party. They were asked to determine how many people could be at the party if there were 4 square tables with a person seated on each side of every table. Students shared their ideas in the following ways.

- **Aidan**—“I did 4 + 4 + 4 + 4 and got 16 people because I knew there would be 4 people and I had to add them 4 times for all of the tables.”
- **Caroline**—“I did 4 + 4 and got 8, then I did another 8 and got 16.”

These two students demonstrated an understanding of the problem as well as reasonable strategies for finding the solution. Caroline found a partial sum and recognized that she could just double it to find the total. Although the procedures were different, both worked effectively to find the solution.

Some third graders explored a problem in which they were asked to determine the number of children at a ball game with 46 girls and 34 boys. Grace determined that there were 80 children at the game and explained her computational method: “I took 40 + 30 and got 70, then I took 6 + 4 and got 10; so 70 + 10 is 80.” While Grace did not do the addition problem using the standard algorithm, her method makes sense and would work with other numbers. In fact, when asked to add two-digit numbers, many people would use a method similar to Grace’s because they find it easier than the traditional algorithm they were taught.

As students are given opportunities to invent procedures to solve problems, they explore numbers, discuss ideas, and often recognize other ways to do them. Through teacher-facilitated discussions, students examine their ideas and further develop their concepts about numbers and operations. They ponder whether their method will always work or whether another students’ method might be easier or more efficient. In traditional algorithms, we often ask students to memorize pro-
cedures that make no sense to them. They memorize words about placeholders or trading or carrying or cross-multiplying, but the words are not connected to a real understanding of numbers. Allowing students to explore problems and invent procedures will help them build understanding; it promotes reasoning rather than memorizing.

Students do benefit from learning traditional algorithms—as one way to achieve computational fluency—as well as from being allowed to invent different procedures. While students may begin by solving problems in nonroutine ways to make sense of math processes, the teacher can use students’ ideas as a starting point for examining traditional methods. As students explore math through problem situations and focus on understanding math processes, they are building number sense, developing problem-solving skills, and arriving at a better understanding of computational procedures.

**How Can Teachers Support Students’ Understanding of Numbers and Operations?**

The foundation for understanding numbers and operations begins in primary classrooms with the concept of counting. Rather than just saying and writing numbers, students need to understand those numbers. Having students explore four objects in a variety of ways helps them understand the concept of “four.” Having them compare four objects to groups with more or less objects, or line up objects to determine one-to-one correspondence, contributes to their understanding of numbers. Older students continue to develop their number sense as they explore connections between numbers through discussions of more than, less than, same as, twice as much, three times more, and so on.

Exploring patterns greatly supports students’ understanding of numbers and operations. Skip-counting helps students recognize and understand patterns and helps strengthen their number sense. As students practice a variety of ways to skip-count (e.g., by 2s, 3s, 4s, 5s, 10s, etc.), they familiarize themselves with the patterns in our number system. Teachers can help make them visible to students by using 100 Charts or number lines (see Appendix C) to show the patterns as students skip-count.

Many teachers capitalize on opening-of-the-day calendar activities to explore numbers and patterns. By placing calendar dates on circles, squares, or triangles, students can explore patterns throughout the month, or by discussing yesterday’s date, what the date is in three days or next Tuesday, students begin to recognize patterns and strengthen their understanding of numbers. While such activities are more prevalent in primary-grade classrooms, many intermediate teachers are modifying the technique to use calendars to highlight multiples or more intricate patterns.

It is important that students first understand the mathematics before they are asked to memorize facts and procedures (Burns 1992a, Van de Walle 2004). Beginning basic computational lessons with problem situations helps set a context for the computations and allows students to hear scenarios to illustrate the operations. Reading children’s literature that illustrates various operations sets a context for learning and understanding those operations. Connecting facts and procedures to real-world activities gives students a real context in which to understand math. Students might add the number of runs in each inning to find a baseball score, or subtract the cost of admission to the zoo from the money in their pockets, or multiply the cost of lunch by the number of students ordering lunch today, or divide the pieces in the manipulative bucket between the students in a group to determine how many pieces each person will receive.

Mental math activities help students develop number sense and provide practice in applying basic skills. While waiting in line for lunch, students might be asked to mentally
compute sums or products. Teachers might verbally pose computation chains for students to solve (e.g., \(3 \times 5 + 2 - 1\)) to stimulate their mental math skills. Teachers might pose story problems for students to mentally compute. Estimation activities provide opportunities for students to demonstrate their understanding of mathematics, as well as opportunities for teachers to help modify those understandings through discussions about how they arrived at the estimates.

**How Can Teachers Help Students Develop Strategies to Understand and Retain Basic Facts?**

Students who use strategies combined with memorization will have a better foundation for understanding and retaining math facts. Prior to memorization, students need to explore problem situations to experience the operations, as well as see visual models to help them more fully understand each operation. Students should be encouraged to discuss the operations and explore strategies to help them understand the process and figure out an answer.

To understand the concept of addition, students should be supported as they explore the concept of joining two sets. A common strategy for finding answers to addition problems is counting on (e.g., the answer to \(5 + 2\) can be determined by counting on to 5 by saying 5, 6, 7). Students should also be supported in seeing turn-around numbers—the commutative property—through many examples to illustrate that \(4 + 2\) will be the same as \(2 + 4\). Still another way that students find addition answers is through using what they already know to figure out unknown equations. Beginning with doubles (e.g., \(2 + 2, 3 + 3\), etc.) gives students a foundation so that they can figure out other sums based on their knowledge of doubles. If \(2 + 2 = 4\), then \(2 + 3\) must be 4 plus 1 more. Helping students use facts they know to find answers to unknown facts provides them with a useful strategy that can be applied to other operations.

The concept of subtraction can be viewed in several ways. The idea of take away is one way to view subtraction, but subtraction is also the operation used when items are compared (e.g., Kate had 6 pieces of bubble gum. Jason had 8 pieces of bubble gum. Who had more bubble gum?). Lining up objects to show the act of comparing will help students see this subtraction concept as they formulate \(8 - 6 = 2\) to find the difference between the two rows of objects (see Figure 2.1).

Another model for subtraction is the missing addend model. Often, we use the phrase “how many more” to illustrate this way of viewing subtraction. For example, Brendan had 4 tickets for the rides. He needed 7 tickets to ride the roller coaster. How many more tickets did he need? You can think of this problem as \(7 - 4\), but often it is thought of as \(4 + ? = 7\) (i.e., 4 plus how many more will give me 7). Discussions about similar problems will help students see this type of problem with more clarity. The various models of subtraction should be reviewed through manipulative activities and story problems, or explored through literature, so that students develop an

---

**FIGURE 2.1** This primary student uses linking cubes to compare groups of objects as she explores subtraction concepts.
understanding of the concept of subtraction. When helping students explore strategies for finding answers to subtraction problems, it is helpful to relate subtraction to addition through fact-family explorations.

The concept of multiplication can be illustrated through manipulative demonstrations or student explorations that show the joining of equal sets. Arrays and area grids, such as those in Figures 2.2 and 2.3, also illustrate the concept of multiplication. In exploring multiplication strategies, students often equate it with the idea of repeated addition. To help them conceptualize multiplication facts, students can be asked to connect facts to real situations. Direct students to look at the 5 fingers on their hands—how many fingers are on 2 hands? Or 5 might be viewed as how many cents there are in a nickel, so how many cents are in 3 nickels? Three might be seen as the wheels on a tricycle—how many wheels are on 2 tricycles?

As students begin to struggle to memorize multiplication facts, it is important to facilitate discussions about strategies for finding an answer prior to asking them to memorize. Encourage students to start with facts they know and to build from there. In multiplication, students discover that multiplying by two is like doubling. Once they know the two times tables, they recognize that fours are just the twos doubled. Students recognize the fives by thinking about familiar counting sequences (e.g., skip-counting). If they know doubles (e.g., $6 \times 6 = 36$), then $7 \times 6$ is 1 more 6, so it is 42. Or some students might break down $6 \times 5$ as a set of $2 \times 5$ and $2 \times 5$ and $2 \times 5$. Through these insights, students are exploring the concept of multiplication as they are developing valid strategies to find solutions.

Understanding division requires understanding the concepts of repeated subtraction and fair sharing. Again, the use of stories or manipulatives helps students see repeated subtraction. For example, Allison has a bag of 21 pieces of candy; she gives one child 7 pieces, another child 7 pieces, another child 7 pieces, and is then out of candy. She has subtracted 7 from her bag of candy 3 different times. Or think about Pat who shares a tray of 12 cookies with 3 friends; this helps students explore the fair-sharing model. As he sorts the cookies into fair (equal) groups, he is illustrating the concept of division. Students can be asked how many sets of cookies will be created or how many cookies will be in each set. Memorizing division facts may be unnecessary because students generally view division as a missing factor equation. Helping students see that 12 divided by 4 is the same as $4 \times ? = 12$ gets them to use the multiplication facts they know to solve division problems.

Keep in mind that discussions about basic operations should not be limited to the primary grades. As numbers become more complex and students are challenged with problems that require computations with fractions, decimals, percents, or large numbers,
they will benefit from discussions about problem scenarios and reflections on the concepts of the operations. The sight of fractions or decimals often causes anxiety that can block students from seeing the problem situation. Replacing more complex numbers with simpler ones, and discussing the scenario using the simpler numbers, generally helps students identify the operation that is reflected in the problem situation. Discussing the concepts behind basic operations should be a routine activity at all elementary grade levels.

**Should Students Be Asked to Memorize Basic Facts?**

Memorization of math facts is necessary to facilitate mental math, to provide students with fluency during computations, and to move easily through problem situations. But, memorizing numbers and symbols without a basic understanding can be very difficult for many students. There are a variety of ways to reinforce and practice facts; however, practice and memorization should occur after students have explored the concepts for the operations.

While many textbooks and curricula push students to master basic facts in a chapter approach (e.g., a unit on basic multiplication facts that asks students to master all facts over several weeks’ time), there is tremendous value in repeated practice over extended periods of time. Students need continued practice to commit basic facts to memory, regardless of the grade level. Rather than lengthy drills, many teachers have seen the benefits of short, engaging practice sessions spanning the school year. Flash cards are a tried-and-true practice activity, but there are many other activities that engage students in the practice of basic facts, from card games to bingo to memory games to sorting activities (see Figure 2.4). At all grade levels, fun and engaging fact practice provides support to students so that retention of facts is possible. For some fun math fact activities to help students review and retain basic facts, see Appendix B.

![Students practicing basic facts](image)
The National Council of Teachers of Mathematics (2000) supports the use of calculators, particularly in problem-solving situations; they help students solve problems that may be beyond their current computational skills. Calculators also help students explore math operations as they investigate patterns and operations by looking at and discussing the results generated by them. The NCTM views the calculator as a tool for problem solving or computational explorations, but recognizes that for basic facts practice, their use may not be appropriate (2000). The choice of when to allow use of calculators in the classroom is an instructional decision based on your math objectives for the lesson. If the purpose of the lesson is to provide students with practice in computations, then the use of a calculator would not support your goal.

**How Can Teachers Assess Students’ Understanding of Numbers and Operations?**

Assessing students’ understanding of numbers and operations goes deeper than checking for correct answers. While answers always provide some information about student understanding, our goal is to determine if students have developed number sense as well as an understanding of operations and computational procedures. Various assessment techniques will provide that information. Observations of student work and attention to their comments inform us about their level of understanding. Informal student interviews (e.g., a series of questions probing their ideas) also can be an invaluable tool. Tasks in which students are asked to draw pictures or write stories to illustrate equations offer insight into their understandings. Having students write an explanation of how they did a procedure (e.g., adding fractions with unlike denominators) helps us see their thinking as they describe how they approached the task. For some ideas about writing descriptions of math tasks, see Appendix H.

Although tests of math facts are used frequently to assess students’ mastery of the basics, timed ones are discouraged because they contribute to students’ anxiety and often undermine their motivation to learn facts. Rather than students competing with each other, many teachers have students chart their own progress (e.g., facts known sheets); they can keep track of their individual achievements and develop pride in their own growth.

---

**HELPING STUDENTS UNDERSTAND BASIC FACTS AND COMPUTATIONS**

**CLASSROOM IDEAS**

**Quick and Easy Computation Activities**

**Do Not “Learn ’em and Leave ’em.”**

Try not to move too rapidly through initial attempts at memorizing math facts. Even if students appear to know the facts during the “multiplication unit,” they will forget them without continued practice. Make facts practice a part of the daily routine through warm-up and closure activities, centers (see Appendix K), or partner reviews. Even activities as simple as frequent two-minute flash card reviews will help students maintain skills.

**Pinch Cards**

Pinch cards are an all-pupil response technique in which each student is given an index card that displays grade-level appropriate operation signs. Second graders might have pinch cards that display addition and subtraction signs, while fourth or fifth graders...
might have pinch cards displaying all four operations. The teacher poses a word problem and students pinch the part of the card that indicates the operation that should be used to solve the problem (see Figure 2.5). This activity provides students with practice listening to and analyzing problem situations to determine the related operation, affords an opportunity for informal discussions about operations, and provides teachers with a quick check on students’ understanding (O’Connell 2000).

Triangle Flash Cards
Working on the missing-factor multiplication model may be more useful to students than drilling division facts because that is how most of us view division problems. With triangle flash cards, the factors and product are placed on the three corners of the card (see Appendix B). The factors are written in blue and the product in red. The teacher, or student partner, covers one number and the student has to find the product or missing factor. Triangle flash cards provide practice with multiplication and division facts and help students see division as missing-factor multiplication.

Variation: Create triangle flash cards to review addition and subtraction. Addends should be written in blue and the sum in red. As one number is covered, the student needs to add to find the sum (if the numbers are both blue) or subtract to find the missing addend (subtract the blue from the red).

Less Is More
Modify the quantity of calculations by cutting the paper in half, crossing off every other problem, or circling only the problems students are to complete. Do fewer computations with more talk about those computations. Rather than just doing them, discuss the how and why of the computations.

What’s the Question?
Provide students with an answer and ask them to come up with a possible equation that would result in that answer. If the answer is 25, students can generate equations such as the following: $10 + 15 = 25$, $5 \times 5 = 25$, $100 - 75 = 25$, $24 + 1 = 25$. For older students, use more complex numbers such as $4 \frac{1}{2}$ or 1.45.

Variation: Students can be asked to write story problems that would result in that answer. Answers might include 25%, 4 pounds, 16 feet, or $2 \frac{1}{2}$ candy bars. Students should be challenged to justify that the answers make sense for the problems they have created.

Chunking Practice Sessions
When working on computational skills (e.g., adding two-digit numbers, multiplying fractions), consider an alternative to traditional teacher-directed lessons followed by lengthy

---

**FIGURE 2.5** As the teacher poses a problem, this student uses a pinch card to indicate the operation she believes will solve the problem.
CLASSROOM IDEAS (continued)

Quick and Easy Computation Activities

practice sessions by chunking the practice into shorter sessions. Rather than a single demonstration followed by extended practice, consider a demonstration followed by a short practice session (maybe three problems). Walk through the room and observe students at work. Use what's observed to then demonstrate the procedure again, talking through your actions. Engage students in discussions to verify their understanding. Then assign two to three problems for student practice and rotate through the room to observe them again. This modified whole-class approach will provide shorter practice sessions that will hold their interest, and it allows you to catch errors before students have committed them to memory through lengthy practice sessions.

Quick Check
This cumulative review technique helps students review and retain previously taught skills. Designate a day of the week (e.g., Tuesday Review) so that it is a regular part of the math routine. Three to five varied computations are posed. Students complete the short activities, then the solutions are reviewed aloud with the class. The teacher uses think-aloud techniques to talk through the solutions or asks students to talk through the solutions.

Summary

Students need to know basic facts. They need to be able to perform math computations fluently to arrive at answers to even simple mathematics tasks. Both invented procedures and standard algorithms have a place in the classroom. By learning computations through models, discussions, problem scenarios, demonstrations, and investigations, students are better able to understand, retain, and apply facts and procedures to real situations and/or more complex math tasks.

Suggested Resources


Questions for Reflection

1. How do invented procedures support students’ understanding of numbers and operations?
2. In what ways can teachers provide repeated, but engaging, practice with basic facts?
3. What is the purpose of timed math fact tests? What are the drawbacks? In what ways can facts be reviewed without those drawbacks?
4. In what ways can you support students who struggle with memorizing basic math facts?


Thank you for sampling this resource.

For more information or to purchase, please visit Heinemann by clicking the link below:


Use of this material is solely for individual, noncommercial use and is for informational purposes only.