I discovered that the number of stools has to be odd, because any number of tables will have an even number of legs. It takes an odd number to add to an even number to get an odd sum of 31.

Strategy 4 — Lists of multiples, with Extra

I listed all the multiples of 4 and 3 up to 31. I looked for pairs which added to 31. I realized that only odd multiples of 3 would work, so I crossed out the even multiples.

Multiples of 4	4	8	12	16	20	24	28			
Multiples of 3	3	~~6~~	9	~~12~~	15	~~18~~	21	~~24~~	27	~~30~~

The combinations that worked were
4 and 27, 1 table and 9 stools
16 and 15, 4 tables and 5 stools
28 and 3, 7 tables and 1 stool

Extra: For each combination above I multiplied the number of tables by $3 and the number of stools by $2. Wendy would get the most money from 7 tables and 1 stool.

Tables	Table $	Stools	Stool $	Total
1	$3	9	$18	$21
4	$12	5	$10	$22
7	$21	1	$2	$23

Teaching Suggestions

Solving this problem involves two kinds of understanding:

1. The constraints of the problem – including the fact that Wendy wants to use all 31 of the legs, and that she'll construct 4-legged tables and 3-legged stools with them.
2. What constitutes a solution – in this case finding all three solutions and demonstrating why there are no more, whether directly stated or implied by the solution strategy.

Some students will use a guess-and-check strategy. Younger students will likely use a manipulative to test numbers. Using 31 items, e.g., toothpicks or cubes, to model the legs is a useful way to solve the problem. Triangles and squares, as in Pattern Blocks, could also be used. Some children will draw pictures or diagrams to model the story. Making a table to keep track of test results is an excellent strategy, since it leads the solver to see patterns and discover insights, especially if the student tests numbers in a systematic way.

Older students may take advantage of their knowledge of multiples to arrive at a solution more directly. Students who have learned how to write expressions with variables might bring that skill to the problem.

Wooden Legs offers an opportunity to make students more aware of the role of parity (odd vs even) in problem solving, as illustrated by strategies 3 and 4 in the Expected Solutions above.

In any case, encourage students to find a representation that makes sense to them. Sharing and discussing strategies after solving the problem may help children who are ready to move forward in their thinking to have an "Aha!" moment, which they can apply to future problems.

The Teacher Support Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

Scoring Rubric

On the last page is the **problem-specific rubric,** to help those who are assessing student solutions. It specifies what we expect from students in three areas of problem solving and three areas of communication. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on every problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

Sample Student Solutions

Focus on Strategy

I've chosen the samples below to illustrate a range of strategies demonstrated by submitters. This category of our rubric addresses whether the student has approached the problem with a sound method and in a systematic way, relying on skill and not luck. A guess-and-test strategy must involve good reasoning and informed guessing. Those who solve the problem more directly need to use valid processes based on well-grounded concepts. In this problem the chosen strategy must allow the child to be confident that she/he has found all possible solutions.

It is not my purpose here to give a definitive judgment, but rather to highlight the range and variety of work done by students and suggest ways I might encourage them to take next steps. The best way to elicit more writing from students is to ask them specific questions about the missing details.

Kira
Age 10

Strategy
Novice

All the ways that you could make are 1) Four tables and zero stools. 2) Seven tables and zero sools. 3) Zero tables and 12 stools. THe anwser for extra credit is #1.

I solved the problem by grouping. I first wrote down 31 lines. Then I grouped then together and added them up to the number of tables and stools.I then had to check to make sure that they were right. Like the first one I would have to do four times nine and get 36 now thats over so you have to change your anwser to four. If you do the second one then it would be 3 times 9 and get twenty seven. There for you have to change your anwser to 7.

Drawing 31 legs is a good first step. It's difficult to imagine how Kira grouped them, or why she checked them by multiplying by 9. This strategy clearly "didn't work." None of Kira's three solutions use 31 legs. I'd ask Kira to paraphrase the problem to check for understanding, and then to show drawings of her solutions to see if she can find her own errors.

Kenneth
age 9

Strategy
Apprentice

1:The ways are 7 four legged tables and 1 three legged stool, 9 three legged stools and 1 four legged tables, and 4 four legged tables and 5 three legged stools.

Extra: The most money she could make is with the 7 tables and 1 stool.

For 1 I played with the number 4 and the number 3 until I got my answers. For the extra I counted the different kinds of tables and stools she could have had and added up those amounts.

Kenneth's correct answer indicates that his strategy was sound, but he doesn't give evidence of what he tried and how he knows he's found them all. Guess-and-Test approaches must involve good reasoning and informed guessing. I'd use the prompts in the answer check to move him forward.

Ryan
age 11

Interpretation
Apprentice

There are three possible answers. 4 tables, 5 stools. 1 table, 9 stools.

7 tables, 1 stool.

I charted out the possible ways to make the chairs and tables. Then I tried making the right amount. I randomly put down four tables (16 pegs) then I did 31-16=15 abd that is divisible by three!

So five stools and four tables is one possible way. Then I saw another one!

Nine stools and one table. I put it on the chart.

The I found one stool and seven tables. I couldn't find anymore possibilites.

For the extra I found that the most money she could make was by selling

1 stool and seven tables for a total of $23.

Ryan's solution is also correct. Looking for multiples of three is a sound approach, but he appears to depend too much on luck. By approaching the problem "randomly" and just trying to "see" combinations that work, he can't be sure he has found them all. I'd encourage him to make a table to organize what he's found so far, and use that to explore whether there can be other solutions.

Taylor
Age 9

Strategy
Apprentice

She can make 5 stools and 4 tables.

I first made 31 legs. Then I circled 3 than 4 than 3 than 4 ... Then I got 5 stools and 4 legs. Then I added 4+5= 9 So she can make 9 items.

Extra: $2.00x5=$10.00
$3.00x4+$12.00
$12.00+$10.00=$22.00

but if she wants more money she would do
$3.00x5=$15.00
$2.00x4=$8.00
$15.00+$8.00=$23.00

So if you multiply the more stools to the highest price you will get more money than doing the tables times the highest amount of money. She will also get more money if she buys more legs and sells twice the much and get twice as much money.

Taylor either missed, or didn't understand, that he was to "Find all the possible ways she can use all 31 legs." ***His Extra is more evidence that he didn't understand that.*** *Taylor's strategy of alternating was an effective way to find one solution, but it* ***"isn't enough to solve the whole problem." Since he seems to understand the key math ideas of the problem, asking him to read the problem aloud could allow him to realize the extra requirement****.*

Kate
age 9

Strategy
Practitioner

The different possibilites I found were 1.7 tables and 1 stool, 2.1 table and 9 stools, and 3.4tables and 5stools.

I went through all 3 and 4 times tables that were under 31 and took the number 4 or 3 was multiplied by and said that's how many tables I made or that's how many stools I made then took the answer to that problem and subtracted it from 31. If the answer I got was divisable by 3 or 4 I knew it could be a complete table or stool using all the legs without leftovers.

"all the 3 and 4 times tables" indicates a systematic search that led Kate to find all solutions. Her description of using subtraction and then testing for divisibility of the remainder is further evidence of her systematic mindset. Her explanation would be more complete if she showed some of the math she described. She might need help in creating a number model.

Emma
age 14

Strategy
Practitioner

The different ways that Wendy can build dollhouse furniture are:
4+4+4+4+4+4+4+3=31* 7 tables,1 stool
4+4+4+4+3+3+3+3+3=31*4 tables, 5 stools
3+3+3+3+3+3+3+3+4=31*1 tables,8 stools

I got these answers by doing this;
I started by adding only fours. I saw that the closest I could get to 31 was 28 without going over. I figured, hey, lets take away a four, and put in a 3. From that I got 31. I continued this process by doing this:
4+4+4+4+4+4+4+3=31, check
4+4+4+4+4+4+3+3=30, nope
4+4+4+4+4+3+3+3=32, nope
4+4+4+4+3+3+3+3+3=31, check
4+4+4+3+3+3+3+3+3=30, nope
4+4+3+3+3+3+3+3+3=29,nope
4+3+3+3+3+3+3+3+3=31, check
3+3+3+3+3+3+3+3+3+3=30, nope.
These are all of the possible answers.

Extra
The scenario containing the most tables is the one with 7 chairs and one stool, making $31!!

*Emma gradual method, starting with the maximum number of tables and decreasing one at a time is very systematic. I would ask her to check her 1-table solution for accuracy. That error could be avoided by an alternate equation, e.g., (1 * 4) + (9 * 3) = 27. She could improve her Strategy to Expert by explaining her Extra method and showing the math.*

Archit
age 11

Strategy
Expert

Answer: The possibilities are:

TABLES|STOOLS
----
1 | 9
----
4 | 5
----
7 | 1

To solve this, I used the following equation:

(31 - 4* amount of tables)/3 = amount of chairs.

If you fill in the value for amount of tables, a chart will look like this:

Tables|Stools
1|9
2|7 2/3
3|6 1/3
4|5
5|3 2/3
6|2 1/3
7|1
8|-1/3

As you cannot have a negative amount of stools, and you are trying to make only whole stools, the only possibilities are 1 table and 9 stools; 4 tables and 5 stools; and 7 tables and 1 stools.

Extra: If you try the values:
With seven tables and one stool she makes $23.
With four tables and five stools she makes $22.
With one table and nine stools she makes $21.
So the highest money will be made by selling seven tables and one stool.

Archit's approach is very sophisticated for his age, as well as very algebraic. He generalized a formula, and then evaluated it for each number of tables, producing a corresponding number of chairs. He is ready to learn about spreadsheets! He exhibits good number sense to determine which of the combinations is a true solution. I'd ask to see the math he used to solve the Extra.

We hope these packets are useful in helping you make the most of FunPoWs. Please let me know if you have ideas for making them more useful.

https://www.nctm.org/contact-us/

Wooden Legs

Wendy builds wooden dollhouse furniture. She uses the same kind of legs to make 3-legged stools and 4-legged tables.

She has a supply of 31 legs and wants to use them all to make stools and tables.

Find all the possible ways she can use all 31 legs.

Explain how you solved the problem and how you know you have found all solutions.

Extra: Wendy sells her furniture to the local toy store. She gets $2 for each stool and $3 for each table. Of all the ways you found, which would earn her the most money?

Be sure to explain how you know.

The Math Forum's Problems of the Week provide non-routine constructed response problems. The Math Fundamentals problems target concepts typically learned in grades 3-5. Memberships and mentoring options are available at the student, teacher, school, and district levels.

Math Fundamentals Problem of the Week Scoring Rubric for Wooden Legs

For each category, choose the level that *best describes* the student's work.

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Understands few of the criteria listed in the Practitioner column.	Shows some understanding of the math in the problem. Completes part of the problem.	Understands that each solution must use all 31 legs. Attempts to find all possible ways to use them. Each stool must have 3 legs; each table must have 4. Answers the main problem.	Solves the main problem correctly. Understands and solves the Extra. Achieves at least Practitioner in Strategy.
Strategy *(Note: based on the solver's interpretation of the problem)*	Does not know how to set up the problem. OR Shows no evidence of strategy. OR Strategy didn't work.	Tries a strategy that makes sense, but isn't enough to solve the whole problem, OR doesn't apply it systematically. OR Strategy is not evident. Might verify a correct answer, but fail to explain how they found it.	Picks a sound strategy. Approaches the problem systematically, achieving success through skill, not luck. Guess-and-Check approach must involve good reasoning and informed guessing. Chosen strategy accounts for any answer(s) that changed after checking our answers.	Does **one or more** of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., effective and appropriate use of technology or algebra.
Accuracy *(Note: based on the chosen strategy)*	Has made many errors. OR Shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic or record keeping mistakes.	Not available for this problem.
Communication				
Completeness *(Note: an incorrect solution can be complete)*	Writes very little to explain how the answer was achieved.	Provides explanation but does not include calculations; OR Shows calculations without explanation about why they were done.	Explains most of the steps taken to solve the problem and the rationale for them, with enough detail for another student to understand. Includes key calculations with rationale. Shows how she/he knows all solutions have been found. Explanation accounts for any answer(s) that changed after checking our answers.	Explains strategy for Extra. Does **one or more** of these: Includes useful extensions and further explanation of concepts or patterns. Provides exceptional insight into the problem. Includes a table of data.
Clarity *(Note: incomplete and incorrect solutions can be explained clearly)*	Explanation is very difficult to read and follow.	Explanation isn't totally unclear, but another student wouldn't be able to follow it easily. Spelling errors/typos make it hard to understand.	Attempts to make explanation readable by a peer. Show critical calculations. Uses level-appropriate math language, including correct units: legs, tables, stools. Shows effort to use good formatting, spelling, grammar, typing. Errors don't interfere with readability.	Formatting makes ideas exceptionally clear. Answer is very readable and appealing, might include a helpful diagram or image. (A diagram or image alone doesn't qualify for Expert status.)
Reflection *(Note: see the items in the gray box)*	Does nothing reflective.	Does one reflection.	Does two reflections.	Does three or more reflections or does an exceptional job with two.
	The items in the columns to the right are considered reflective, and might be in the solution or *comment:*	• **Revises and improves a previous submission.** • Checks the answer using a different method. • Explains a hint she/he would give another student.	• Reflects on the reasonableness of the answer. • Connects the problem to prior knowledge/experience. • Describes any errors made and how she/he found and corrected them. • States any assumptions made in the solving process. • Described something learned from the problem.	• Comments on *and* explains the ease or difficulty of the problem. • Explains where she/he is stuck. • Summarizes the process used. • Describes any "aha!" moments.