# Math Fundamentals PoW Packet

# *Marble Mayhem*

**Problem 4663 • https://www.nctm.org/pows/**

**Welcome**

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in February 2008. The text of the problem is included below. A print-friendly version is available using the "Print" link on the problem page.

**Standards**

In ***Marble Mayhem*** students are asked to find the fewest number of marbles that Fred could have had in his bag. This problem requires understanding the concepts of division as equal sharing, of multiples as repeated addition of the same number, and of common multiples. Young children without experience with the formal vocabulary and procedures involved can successfully solve the problem by using a systematic guess-and-check strategy.

If your state has adopted the Common Core State Standards, this alignment might be helpful.

*Grade 3: Operations & Algebraic Thinking*

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

*Grade 4: Operations & Algebraic Thinking*

Use the four operations with whole numbers to solve problems.
Gain familiarity with factors and multiples.
Generate and analyze patterns.

*Mathematical Practices*

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
8. Look for and express regularity in repeated reasoning.

**The Problem**

***Marble Mayhem***

Fred, Ginger, Julio and Dawn decided to play marbles. Fred emptied his bag of marbles and divided them equally among the four players. Everyone got at least one marble. There was one marble left over.

At that moment Jake arrived and asked to play. They gathered up all Fred's marbles and divided them equally among the five kids. There was still one marble left over.

Just then Maria joined them, so they gathered all the marbles again and divided them equally six ways. There was still one marble left over.

What is the fewest number of marbles that Fred could have had in his bag?

**Extra**: What is the fewest number of marbles Fred could have had in his bag if Dawn had not been there at all? How did your answer compare with your original answer? Why do you think that is so?

## Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

The smallest number of marbles that Fred could have had in his bag is 61.

If your answer **doesn't** match ours,

- did you understand that each time the marbles were divided, each child got an equal share?
- did you remember that there was always one marble left over after dividing them equally?

If your answer is greater than 61, did you remember to look for the smallest number that would work?

If you used guess-and-check, did you tell . . .

- what numbers you tried?
- how you checked them?
- how you knew whether they worked or not?
- how you decided what to try next?
- about any patterns that helped you?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're **still stuck**, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- have you clearly shown and explained the work you did?
- did you make any mistakes along the way? If so, how did you find and fix them?
- are there any hints that you would give another student?
- does this problem remind you of experiences you've had?
- did you try the Extra?

*Revise* your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

## Our Solutions

**Method 1: Using Manipulatives:**

I used inch tiles to represent marbles. When you count by fives, every number ends in zero or five, so the answer has to end in one or six, in order to have one left over. Since I needed the smallest number that works, I started with six tiles and tried to put them in four equal groups, with one left over. It didn't work.

Then I figured out that any number that makes four equal groups has to be an even number. To have one left over, the answer has to have a one in the one's place. I tested 11, but it did not work because you can't make four equal groups out of ten. I could divide 21 into four groups and five groups with one left over, but not six equal groups.

I continued testing numbers that end in one.

- 31 didn't work for four groups.
- 41 worked for four and five, but not six groups.
- 51 didn't work for four groups.
- 61 worked!

I could divide 61 into four, five and six equal groups with one left over each time. It is the smallest number of marbles Fred could have had in his bag. Four children each would have gotten 15 marbles (4 • 15 = 60). Five each would have had 12 marbles (5 • 12 = 60). Six each would have had 10 marbles (6 • 10 = 60).

**Extra:** If Dawn had not been there, Fred still had to have 61 marbles in his bag for the story to work. I had to find the smallest number that I could divide into three, four and five equal groups with one left over. It still had to end in one because of the four and five equal groups. I thought the answer would be smaller, but I when I used the tiles to test, I found out that 61 was the smallest number that would work. With three children each gets 20 marbles (3 • 20 = 60).

**Method 2: Skip Counting**

I knew that the answer had to be one more than a number that had factors of four, five and six. I made a list of numbers you land on when you skip count by fours, fives and sixes. Here they are:

| 4s | 5s | 6s |
|---|---|---|
| 4 | 5 | 6 |
| 8 | 10 | 12 |
| 12 | 15 | 18 |
| 16 | 20 | 24 |
| 20 | 25 | 30 |
| 24 | 30 | 36 |
| 28 | 35 | 42 |
| 32 | 40 | 48 |
| 36 | 45 | 60 |
| 40 | 50 | 66 |
| 44 | 55 | 72 |
| 48 | 60 | |
| 52 | 65 | |
| 56 | 70 | |
| 60 | 75 | |
| 64 | 80 | |
| 68 | | |
| 72 | | |

I began looking down the list of 6s, because there are fewer of them to check. I looked for a number that appeared in all three lists, because that number would have all three factors. Many numbers appeared in two lists but not in all three until I reached 60! It is the smallest number that has factors of 4, 5, and 6. The answer has to be one more that that, or 61.

**Extra:** If Dawn were not there I would have to find the number that appeared in the lists of 3s, 4s, and 5s. I already had skip counted by 4s and 5s, so I added the list that counts by 3s:

3s: 3, 6, 9, 12, … 54, 57, 60

I was surprised that 60 was the smallest number that appeared in the list of 3s, 4s, and 5s. Fred would have needed 61 marbles in his bag in this case also.

**Method 3: Least Common Multiple**

Since the answer, less the one marble left over each time, can be equally divided into 4, 5 and 6 groups, the answer has to be one greater than the least common multiple of 4, 5, and 6. I did a prime factorization of all three numbers:

4's prime factors are 2 • 2.

5's prime factor is 5.

6's prime factors are 2 • 3.

The smallest number that has all those factors is 60. Its prime factors are 2 • 2 • 3 • 5. It contains all the prime factors of 4, 5, and 6. Every time the kids divide the marbles, there is one left over, so the bag must have started with 61 marbles.

**Extra:** If Dawn was not there, the problem becomes finding the least common multiple of 3, 4 and 5. That is still 60. Fred would still need to have 61 marbles in his bag at the start.

## Teaching Suggestions

Children who are developing multiplication concepts will be able to apply them to *Marble Mayhem***.** As suggested in Method 1, manipulatives can be used to model the problem. Children with more developed concepts of multiples might mark the multiples of 4, 5, and 6 on a 100 grid, using a different color crayon or chip for each multiple. Asking students questions that elicit their understanding about multiples of 5, odd/even numbers, and units digits can help them develop insights that lead to a more efficient approach.

Some students may list numbers that are multiples of 4, 5, and 6 and then add 1 onto the least common multiple, demonstrating good number sense in terms of the affect of the left over marble. Others may list the numbers that are one more than the multiples of 4, 5, and 6, (e.g., 5, 9, 13, 17 for the 4s) and then look for the common number in these lists. While this is a sound strategy, the numbers in these lists are less "friendly" and make it more difficult for students to make use of what they know about the patterns in multiples and final digits. Careful questioning may help move these students toward a more efficient approach.

## Sample Student Solutions

focus on **Interpretation**

In the solutions below, we've provided the scores the students would have received in the **Interpretation** category of our scoring rubric. Our comments focus on what we feel is the area in which they need the most improvement.

| Novice | Apprentice | Practitioner | Expert |
|---|---|---|---|
| Understands few of the criteria listed in the Practitioner column. | Understands most but not all of the criteria listed in the Practitioner column. | Understands the<br>• problem asks to find the fewest number of marbles Fred could have had in his bag.<br>• division concept of equal sharing.<br>• multiple concept (repeated addition of the same number.<br>• idea of having one marble left over.<br>• concept of "fewest." | Is at least a Practitioner in Strategy and comes up with the correct solution for the Extra. |

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**Lindsay**
age 11

Interpretation
**Novice**

7 would be the least amount of marbles Fred would have had.6 would be the least number of marbles if Dawn would not have shown up.

On a piece of paper I drew a different face for Fred,Ginger,Julio and Dawn.First i drew a circle under each face to represent a marble. As I read on Jake came so i drew a face for him and drew a marble under his face. When Maria came i also drew a face for her and drew a marble under her. Then i added up the number of marbles under there faces and got 6,then i added the let over marble nad got 7.

*Lindsay has taken pains to write a complete explanation of her strategy. She has missed how the same number of marbles gets redistributed at each stage. I'd ask her to read it again, paraphrase the story, and act it out with manipulatives.*

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**Tim**
age 11

Interpretation
**Apprentice**

There is a total of 61 marbles in Fred's bag.

First i think of how many people there are alotgether. There is six. Now i will guess and check to find my answer. If the four kids got 3 marbles each and there was one left over that would be thirteen marbles. Then Jake comes and divide the marbles with 5 people each person got two marbles theres three left over that does not work. NOw i will try that the first four kids each get 15 marbles. Then Jake comes and each kid get 12 marbles. Finally Maria comes and each kid got ten marbles. That would mean there is 61 marbles total.

*Tim understands the story and division as equal sharing. I'm glad he told how many marbles each child gets with a total of 61. Neither his starting number of 13 or his answer of 61 tell us that he understands the "least" requirement. I'd ask him to show that 61 meets it.*

**Rodney**
age 13

Interpretation
**Apprentice**

The fewest number of marbles that Fred could have had in his bag is 121 marbles.

step 1. since there were first four people, then five, then six, the answer

would have to be a multiple of 4, 5, anf 6.

step 2. i multiplied 4x5x6=120

step 3. then i added one since each time it was divided it there was one left

*Rodney understands the role of a common multiple and knows how to find one. While he uses "fewest" in his answer, his solution doesn't show that he has considered that. I'd ask him how he could find out whether his answer meets that criterion.*

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**Nick**
age 10

Interpretation
**Practitioner**

The leaast amount of marbles that the children can have is 61.

I decided that I needed to know which number 4,5,and 6 could divide into and have no remainder. I then did least common multiples for the 3 numbers.

4 8 12 16 20 24 28 32 36 40 44 48 52 56 60

5 10 15 20 25 30 35 40 45 50 55 60

6 12 18 24 30 36 42 48 54 60

I then took the next number becase each would have a remainder of 1 if divided into 60.

*Nick shows he understands the problem and how finding the LCM will help. By listing all the factors of 4, 5, and 6 in order, he is able to find the lowest common multiple. I'd encourage Nick to try the Extra. He'd be able to take advantage of the work he's done so far, and maybe discover why the LCM is the same.*

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**Teddy**
age 11

Interpretation
**Practitioner**

The fewest amount of marbles Fred could have had in his bag is 61.

First I broke it up into three diagrams. Then I kept on adding marbles to the diagram for when only 4 people were playing until it fit the one where 5 were playing. I did the same with the diagram where 6 people were playing until the same  amount of marbles fit each diagram evenly. My answer was that the fewest amount of marbles Fred could have had was 61.

*I'd like to see Teddy's diagrams. He shows he understands equal sharing. While he doesn't mention it explicitly, his method does address the "least" criterion. I'd ask him to show calculations to verify his answer and tell how many marbles each child got at each stage.*

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**Woodburn TAG Class**
age 9

Interpretation
**Expert**

For #1 The least number of marbles Fred could have is sixty-one.

#2 If Dawn would not have been there, Fred would have had sixty-one marbles because sixty-one is the only number that is the same between the multiples of 3,4,5.

First, we found the multiples of four , five and six because there were that many players.Then we added one because of the one marble that was left over.Thaen we eliminated the even numbers in the multiples of five plus one becase the multples of four and six plus one were all odd numbers.Finnaly, welooked for a number that was common between all the numbers that were multiples.

The multiples of 4 (plus one) were

5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, 77 81, 85, 89

The multiples of 5 (plus one) with the multiples ending in 5 eliminated were

*This class understands all the key ideas of the problem. Their strategy is sound and results in the fewest marbles. The numbers resulting from adding one onto each multiple are not as "friendly" to think about and find patterns. I'd stretch them to think about whether it was necessary to add the one to each multiple.*

11, 21, 31, 41, 51, 61, 71, 81, 91, 101

The multiples of 6 (plus one) were 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73, 79, 85, 91

The lowest number that was common to all of these was 81.

When we found the multiples of 3 (plus 1) they were 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52, 55, 58, 61, 64, 67, 70

Again the lowest common number was 61.

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**Kyle**
age 11

Interpretation
**Expert**

The fewest number of marbles that Fred could have had in his bag is 61.

First there were 4 players, and then became 5 players, and 6 players. So I want to find out the smallest number that is a multiple of 4, 5, and 6. And then you can add left-over 1 to get an answer.

I have to find out the LCM (least Common Multiple) of 4, 5 and 6. Multiply each factor the greatest number of times it occurs in any of the numbers. 4 has two 2s, and 5 has one 5, 6 has 2 and 3.     2X2X3X5=60 ...LSM

I check my work by verifying that 60 can be divided evenly by 4, 5, and 6.

If you add left-over 1 to 60, 61 is my answer.

I check my work by veryfiying that 61 can be divided evenly by 4, 5, and 6, with one left over.

61 / 4 = 15 R1

61 / 5 = 12 R1

61 / 6 = 10 R1

Extra: The fewest number of marbles Fred could have had in his bag if Dawn had not been there is also 61.

I have to find out the LCM of 3,4,and 5.    2X2X3X5=60     And add left-over 1, 61 is my answer.

Both answers are same, because LCMs for both sets of numbers are same.

*Kyle understands all the conditions of the problem. He demonstrates a sophisticated method of finding an LCM, and explains it well. I like the fact that he verified his answer, showing how many marbles each child got. He shows why the LCM for both scenarios is the same.*

**Scoring Rubric**

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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