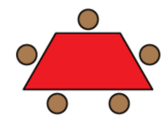
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

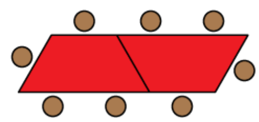
Problem of the Week Teacher Packet

Trapezoid Teatime



Lipton Elementary School holds an annual tea to honor the parent volunteers who work in the school. The trapezoid tables they use can seat one person on each of the three short sides and two people on the long side. In other words, one table standing alone seats five people.

The tables are arranged in one long row in the cafeteria. When they connect two tables together, here's how the seating looks:



1. How many guests can sit at 5 tables connected in a row?
2. How many guests can sit at 20 tables connected in a row?

Explain how you found your answers. Describe any observations or patterns that helped you.

Extra 1: Use words or numbers and symbols to write a rule for calculating the number of volunteers that can sit at any given number of tables.

Extra 2: How many tables would it take, arranged in a straight line, to seat 85 volunteers?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

> *Five tables connected together in a row will seat 17 guests. Now you can figure out how many can sit at 20 tables.*

If your answer **doesn't** match ours,

- did you try using pattern blocks or drawing the tables?
- did you make an organized list to help you see patterns?
- did you check your arithmetic?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer **does** match ours,

- is your explanation clear and complete?
- did you try the Extra questions?
- did you verify your answers with another method?
- did you have any "Aha!" moments or notice any patterns? Describe them.

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Use Manipulatives and Make a Table

I used pattern blocks to solve the problem. First I connected two trapezoids together, as in the picture, and counted 8 seats. Then I added another table and counted 11 seats. I made a list to show the number of tables and the number of seats. I kept adding tables and counting seats until I had 5 tables. Here's my list:

number of tables	number of seats
1	5
2	8
3	11
4	14
5	**17**
...	...
20	**62**

I found there would be 17 seats at 5 tables. I noticed that each time I added a table, the number of seats increased by three. That is because we are adding five new places but losing two on the sides of the tables that connect.

To answer question 2, I counted out 15 more trapezoids to make a total of 20 tables. Then I skip counted by threes for each new table, starting with 17, until I came to 62 seats for the 20th table.

Method 2: Use Virtual Manipulatives and Make a Table

I used the pattern block applet to help me see the pattern. After adding several tables I discovered that each table added three seats to the total. At the ends there were always two more seats, no matter how long the row was. Here are my results:

number of tables	number of seats
1	5
2	8
3	11
4	14
5	17

To answer question 2, I figured out how many more tables I would need: 20 - 5 = 15 more tables

Each of those 15 new tables would add three seats to the row: 15 • 3 = 45 more seats

I added the seats from 5 tables and the new seats: 17 + 45 = 62 total seats at 20 tables

Method 3: Direct Calculation based on seats lost

I multiplied the number of tables by the number of people who can sit at one table:

5 • 5 = 25

For each of the places where two tables connect, we lose 2 seats. There are 4 of those places, one less than the total number of tables.

4 • 2 = 8

I subtracted the number of seats lost from the total places:

25 – 8 = 17 seats at 5 tables

I did the same thing for 20 tables:

20 • 5 = 100 total seats
19 • 2 = 38 seats lost at the connections 100 – 38 = 62 seats at 20 tables

Extra 1: Multiply the number of tables by 3 for the number of seats along the parallel edges of the row. Then add two more seats for the two ends.

OR Start with the number of tables and subtract the two end tables because they each have 4 seats. Multiply the remaining tables by 3 for the seats on the parallel sides, and then add the 8 seats for the end tables.

OR Let n stand for the number of tables. The number of seats will be $3n + 2$. Each table adds 3 seats along the parallel edges. There are two more seats on the ends.

Extra 2: I subtracted the two end seats from the 85 seats needed, because all but two people will sit along the parallel sides of the row.
85 - 2 = 83

Each table seats 3 people along the parallel edges, so I divided 83 by 3. 83 ÷ 3 = 27 r 2

The 2 remaining people need to sit at a table also, so it will take 28 tables to seat 85 guests.

OR From question 2, I knew that 20 tables would seat 62 people. I counted on by 3s. It took 8 more 3s to reach 86, so it will take 28 tables to seat 85 people. There will be one seat left over.

To check my answer I multiplied the number of tables by 3 and added 2 for the end seats.

3 • 28 + 2 = 86

Standards

If your state has adopted the Common Core State Standards, you might find the following alignments helpful.

Grade 3: Operations & Algebraic Thinking

Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations.

Grade 4: Operations & Algebraic Thinking

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Grade 5: Operations & Algebraic Thinking

Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.

Teaching Suggestions

In ***Trapezoid Teatime*** the number of seats is equivalent to the perimeter of the row of tables. While the only arithmetic skills needed to solve the problem are counting and basic operations with whole numbers, the problem develops algebraic thinking by providing a concrete example of a linear function. Each new table added contributes three new seats to the total, analogous to slope, and every stage includes one seat at each end, analogous to the constant or y-intercept. Students ready for extra challenge can graph their results.

Pattern blocks, real or virtual, are good tools for modeling the situation and counting the number of seats. Some students may choose to draw the tables. Those who gradually "grow" the row, counting seats at each stage and keeping track of results in a list or table, will likely discover that each new table contributes three seats to the total. As a table with five places gets added, two seats are lost along the connected edges.

My hope is that students will discover the growth pattern and apply it to calculate the number of seats in the 20-table row, or at least use the pattern to verify what they learn from counting. Students who fail to recognize a pattern can be encouraged through careful questioning to uncover more of the math.

The goal of Extra 1 is to create a rule based on the number of tables (closed form), not simply by adding three onto the number of seats in the previous stage (recursive form).

To solve Extra 2 students can work their observation or rule in reverse. This is an opportunity to explore the idea that, when performing inverse operations, the order of the operations must be reversed. In this case, we subtract the two end seats before dividing by 3. Commend those who recognize that the 28th table is required in order to seat all the people, not merely because 2 is more than half of 3.

Sample Student Solutions – Focus on *Strategy*

In the solutions below, I've provided scores the students would have received in the **Strategy** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has no ideas that will lead them toward a successful solution or shows no evidence of strategy.	Uses a strategy that uses luck instead of skill, or doesn't provide enough detail to determine whether it was luck or skill.	Uses a strategy that relies on skill, not luck, which might include: • thorough noticing and wondering • use logical reasoning • draw a picture • make a table or a list	Does one or more of these: Uses two different strategies. Uses an unusual or sophisticated strategy.

Rachel, age 10, Novice

1. 25 people can sit at 5 tables connected in a row. 2. 100 people can sit at 10 tables connected in a row.

Well, for number 1, I multiplied 5 by 5,and got 25.
And for number 2, i multiplied 20 by 5, and got 100 people. i checked my answer by multipying 20 by 5 on the calculator, and got 100 people once again.

Rachel's answers indicate that she understood the tables were "connected in a row." Her strategy, however, failed to take note of the diagram of 2 connected tables. I'd draw her attention to the diagram and ask if her strategy would result in the number of seats shown. I'd ask her to use pattern blocks to explore adding one table at a time and to keep track of her results.

Jb, age 11, Novice

The answer for 1 is 17 and the answer for 2 is 80.

I got both answers by seeing how many people sit at each table which is 4. Then I multiply by the number of tables and it gives you how many people sit at each table.

Jb's stated strategy was based on the number of seats per table in the 2-table diagram. It doesn't account for his answer of 17 seats at 5 tables or for what happens as each additional table is added. I'd ask him to show how he arrived at 17 and then use pattern blocks to move him toward a better understanding of the pattern.

Rebecca, age 13, Apprentice

17 people will fit at 5 tabels and 80 people will fit at 20 tables.

17 people will sit at 5 tables because 5 people will fit at one table and every table after that you add 3. 80 people will fit at 20 tables because 8 people times 10 tables equals 80 people. It would take 22 tables to fit 85 people because 80 plus 3 is 83 which is'nt enough but add another 3 and you get 86. So you have 1 person left over.

Rebecca described a sound strategy for 5 tables. It appears that she ignored what she learned by simply multiplying the seats in the 2-table diagram by 10, the number of 2-table groups made from 20 tables. I'd want to know how she arrived at her first insight and ask how she might extend that thinking to 20 tables.

Melissa, age 13, Apprentice

For My first answer for the problem is 17 and my second answer is 34.

I got my first answer which was 17 because I found out that there was 5 tables that 2 guests per table can sit at. So I did 5x2 which equalls to 10, then i found out that there was 5 tables that 1 guest per table can sit at plus the two on the side so i did 5+2 which equalls to 7 so to find my answer I did 10+7 which equalls to 17.

I got my second answer which was 32 because I found out that there was 10 tables that had the wide side of the table stick out which 2 guests could sit at per table. So I did 10x2 which equalls to 20. Then I found out that there was 10 tables which 1 guest could sit at per table plus the 4 on the sides. So I did 10+4 which equalls to 14. So I did 20+14 which equalls to 34.

I'm guessing that Melissa either drew tables or used pattern blocks to arrive at her strategy for 5 tables, based on the exposed horizontal edges that seated 1 or 2. Had she applied it accurately to 20 tables, it would have worked. It's possible that asking why she counted 10 of each kind instead of 20 would give her an "aha" moment that would set her back on track.

Christina, age 12, Practitioner

My Five tables connected together would equal 17 guests and twenty tables connected together would equal 62 guests

First I drew the 5 tables connected and realised that 8-5=3; so then for each table I added 3 guests.

1) 5
2) 8

Christina reported drawing the five tables. I trust that her count for the first 5 tables came from her observation of them all and not just the second table, "8-5=3." I'd ask to make

3) 11
4) 14
5) 17
6) 20
. . . *[Christina continued accurately, one table at a time.]* 19) 59

20) 62

sure, and then ask if she can see in that 5-table representation a reason why each table adds 3 guests. Can she use that information to discover a rule? Her list of data could help her test her rule on several numbers of tables.

Annie, age 11, Practitioner

My final answer for number 1 is that 17 people can sit at 5 tables and 62 people can sit at 20 tables.

First, I drew a picture showing how many people can sit at 5 tables. Eight people can sit at the bottom and 7 at the top, plus two people at each of the ends. 7+8+2=17

Then, in order to get twenty, I took 2 off of 17 to keep the two people for later. After, you simply multiply 15x4=60. You multiply by 4 because there are 4 multiples of 5 in 20.

Finally, you add the two for two people on the ends. 60+2=62

Annie counted seats along the two horizontal edges of the 5-table arrangement, and then added the 2 end seats. Her strategy for 20 tables took advantage of the fact that 20 is a multiple of 5. I'd ask her how she would find the seats at 18 tables, helping her to move toward a more general rule.

Sheila, age 9, Practitioner

The answer to problem #1 is 17 people. I got this answer by mutipling 5 tables and 3 people sitting at each table ignoring the two at the end that = to 15 people. Then, you ad 2 that = to 17

The anwser to problem #2 is 62 people. I got this answer by mutipling 20 tables and 3 people sitting at each table ignoring the two at the end that = to 60 people. Then, you ad 2 that = to 62

Sheila was on the verge of describing a general rule. I'd ask her to explain how she made her discovery (drawing? Blocks?) and then model some ways she can represent her steps as number models (equations). I think she is ready for both Extras!

Rohit, age 9, Practitioner

The answer for the 1st question is 17 and the answer for the second queston is 62

I figured the answer for question 1 by adding 5 tables together which makes 25 people if the tables are separate if you combine them there are 4 edges 1 less than the number of tables then 2 people have to go out evey edge so 2x4=8 people have to go out and then 25-8=17 people

I figured the answer for question 2 by adding 20 tables together which makes 100 people people if the tables are separate if you combine them there are 19 edges 1 less than the number of tables then 2 people have to go out evey edge so 19x2=38 people have to go out and then 100-38=62 people

Rohit's strategy involved finding the total capacity of the tables standing alone and then subtracting the seats lost where the tables connect, which is twice the number of connections, which in turn is one less than the number of tables. I'd proceed as for Sheila, helping him create number models and think about the Extras.

Hayley, age 10, Expert

You can fit seventeen people at five trapezoid shaped tables put together. You can fit sixty-two people at twenty trapezoid shaped tables put together. EXTRA 2: It will take twenty-eight tables put together to seat at least eighty-five people.

1: On paper I drew five trapezoid tables put together. I put two seats on the long side, just like the problem said to, and one seat on the short side. Next I put one seat on the ends of the tables, but not in the middle because the tables are pushed together. When I added the seats up I got the sum of seventeen.

2: This time, since I knew at least three people sat at each table, I multiplied three by twenty and got sixty.
3 * 20 = 60

Then I added two more seats because one person sits at each end.

EXTRA 2: I started from twenty tables. I added seven tables, but that only came to eighty-three seats, so I added one more table. That came to eighty-six seats, which is enough to seat eighty-five people.

From her basic counting strategy Hayley discovered a pattern that helped her solve the 20-table problem more easily. For her Extra she uses a guess-ant-test strategy, adding tables according to her pattern and making sure there was a seat for everyone.

Sydney, age 10, Expert

My answer to number 1 is 17 guests. Number 2 is 62 guests. Extra number 1 is 3 x the number of tables + 2. The answer for number Extra 2 is 28 tables.

I figured out number 1 by this formula that I made. You times 5 with 3 and you add 2 because there are 5 tables and 3 seats in each. You add the 2 because you have to count the ends. So 5 x 3 =15 + 2 = 17.

The way to got number 2 is the same way. There were 20 tables and 3 seats in each, not counting the sides. So, you times 20 with 3, which looks likes this (20 x 3 = 60) and plus 2 for the sides, equals 62 guests.

Extra 1 was how I solved all the problems. I times 3 with the numbers of tables and then add 2 and you'll get your answer. For example, there were 3 tables. You times it by 3, which equals 9, and plus 2 equals 12. Extra 2 was hard. I got the answer by minus 85 with 2. That gave me 83. And that was when I almost got the answer. So 3 times whatever the tables are equals 83. Then I divided 83 with 3 and that gave me with 27 R 2. There would be 27 tables , but 2 guests without seats. So I added one more table and I got 28.

Sydney developed a rule from the start and used it to solve both of the main questions. He did a good job explaining his rule in terms of the physical model. I'd model better math language ("multiply" rather than "times", "subtract 2 from 85" rather than "minus 85 with 2"). He recognized the need to reverse the order of his operations in Extra 2 as well as the need to round up to seat all 85 people.

Alexander & Arick, ages 9 and 10, Expert

1.There were 17 people able to sit at 5 tables. 2. There were 62 people able to sit at 20 tables.

First we started out drawing the tables. We figured out that 17 people could sit at 5 tables. After that we drew 10 tables and doubled it to get 64 people at 20 tables. But we figured out that you can't have 4 end tables when there are only 2 ends.

So we subtracted the two people and came up with 62 people at 20 tables.

We had 18 middle tables that seat 3 so its 18*3=54 people.
Then we added 8 people for the two end tables. 54+8=62 people.

That also worked for the first problem. 3*3=9+8=17 people so we know we are right.

These boys applied several strategies. The rule they developed handles the inside tables apart from the two end tables, which seat 4 each. To solve Extra 2 they subtract the 2 end tables, and then try 25, a friendly number to multiply mentally, as their guess. Then they adjust to seat all the guests.

Extra 1: The rule is number of middle tables * 3= people at middle tables + 8 people on both end tables = total people.
Extra 2 There are 28 tables for 85 people.

We used the rule from extra number 1 to help us find the 28 tables.

85-8=77 people at middle tables.
25*3=75 people at middle tables. We knew we needed one more table in the middle and made it 26 tables. Then we added the end tables on 26+2=28 tables. We would have one extra space.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of the Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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