



Pre-Algebra PoW Packet

Kaytee's Contest

Problem 16004 • <https://www.nctm.org/pows/>

Welcome

This packet contains a copy of the problem, the “answer check,” our solutions, some teaching suggestions, and samples of the student work we received in September 2009. The text of the problem is included below. A print-friendly version is available using the “Print” link on the problem page.

Standards

In *Kaytee's Contest*, students are asked to use the clues to figure out how much Kaytee's cows each weigh. The **key concepts** are proportional reasoning, measurement, and possibly an introduction to using variables and setting up and solving an equation.

If your state has adopted the [Common Core State Standards](#), this alignment might be helpful:

Grade 6: Ratios & Proportional Relationships

Use ratio and rate reasoning to solve real-world and mathematical problems.

Grade 7: Ratios & Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

Grade 7: Expressions & Equations

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.

Kaytee's Contest

The Problem

Farmer Kaytee brought one of her prize-winning cows to the state fair, along with its calf. In order to get people to stop and admire her cow, she thought she'd have a “guess the weight” contest, but being mathematically-minded, she added a little twist.

“Guess the weight of Bertha and her baby, Billy! Together Bertha and Billy weigh 1,696 pounds, and Bertha weighs 848 pounds more than Billy does. How much does each weigh?”

Extra: One highlight of many state fairs is the display of cows carved out of butter. According to the [Iowa State Fair website](#), a butter cow Bertha's size weighs 600 pounds and would cover 19,200 slices of toast (which would take a person two lifetimes to consume). How many slices of toast would a butter cow Billy's size cover? How many lifetimes would it take to eat that much buttered toast?



photo by Kaytee Ray-Riek

Answer Check

After students submit their solution, they can choose to “check” their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn’t agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Bertha weighs 1,272 pounds and Billy weighs 424 pounds.

If your answer **doesn’t** match ours,

- did you try making a guess?
- did you notice that Bertha’s weight plus Billy’s weight has to be 1696 pounds?
- did you notice that Bertha’s weight minus Billy’s weight has to be 848 pounds?

If you used guess and check, did you explain:

- what numbers you tried?
- how you tested them?
- how you knew whether they worked or not?
- how you decided what to try next?

If any of those ideas help you, you might *revise* your answer, and then leave a *comment* that tells us what you did. If you’re still stuck, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- did you try the Extra?
- have you clearly shown and explained the work you did?
- did you explain your work as well as you could?
- did you make any mistakes along the way? If so, how did you find and fix them?
- what hints would you give another student?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Guess and Check

I need to find out

- how much Bertha weighs
- how much Billy weighs

I know that Bertha weighs 848 lbs. more than Billy. What if I first guess that Bertha weighs 848 lbs. since she would then weigh that much more than Billy as it states in the problem. Billy would weigh 0 lbs. Wups, that’s way too low!

What if Bertha weighs 1000 lbs.? Billy would weigh 152 lbs. (because $1000 - 848 = 152$) and that’s still too low. I know their total weight is 1696 lbs. This guess of 1000 lbs. is about 500 lbs. too low. Next I’m going to try half of the 500 added to the 1000 guess.

If Bertha weighs 1250 lbs., Billy would weigh 402 ($1250 - 848 = 402$). The total needs to be 1696 but 1250 plus 402 is only 1652 but I’m really close now! Somehow I need Bertha and Billy to each weigh a little more to take care of those 44 lbs.!

If Bertha weighs 1270, then Billy weighs 422 and combined that gives 1692. I’m still 4 lbs. away from having it work. If Bertha weighs 1272 lbs., then Billy weighs 424 lbs. ($1272 - 848 = 424$). Those are the two weights because $1272 + 424 = 1696$!

Method 2: Make a Table Using Bertha’s Weight Possibilities

I know from the problem that Bertha weighs 848 lbs. more than Billy. I also know that their combined weight is 1696 lbs.

If I try a weight for Bertha (list in the first column of my table) I can calculate Billy’s weight (list in the second column). I can combine those two weights (list in the third column) and then compare that to 1696 lbs. to see how close I am. Here’s what I tried:

Bertha's weight (in lbs.)	Billy's weight (in lbs.)	Total weight (in lbs.)	Is total too low, too high ?
848	0	848	848 is 848 lbs. too low
1048	200	1248	1248 is 448 lbs. too low
1248	400	1648	1648 is 48 lbs. too low
1268	420	1688	1688 is only 8 lbs. too low
1272	424	1696	just right!

I know that Bertha weighs 1272 lbs. and Billy weighs 424 lbs.

Method 3: Make a Table Using Billy's Weight Possibilities

I know from the problem that Billy weighs 848 lbs. less than Bertha. I also know that their combined weight is 1696 lbs. If I try a weight for Billy (list in the first column of my table) I can calculate Bertha's weight (list in the second column). I can combine those two weights (list in the third column) and then compare that to 1696 lbs. to see how close I am. Here's what I tried:

Billy's weight (in lbs.)	Bertha's weight (in lbs.) (Billy's weight + 848)	Total weight (in lbs.) (Billy's weight + Bertha's weight)	Is total too low or too high?
200	1048	1248	1248 is 448 lbs. too low
400	1248	1648	1648 is 48 lbs. too low
420	1268	1688	1688 is only 8 lbs. too low
424	1272	1696	just right!

I know that Billy weighed 424 lbs. and Bertha weighed 1272 lbs.

Method 4: Proportional Reasoning and Drawing a Diagram

After reading the problem I know that:

Bertha weighs 848 lbs. more than Billy.
Together Bertha and Billy weigh 1696 lbs.

I notice that the first weight ends in "8" and the second weight ends in "6" and that leads me to confirm that:

$$848 + 848 = 1696$$

So the difference between their weights is half of the sum of their weights. I wonder if I can use that to solve the problem.

I decided to draw a picture to help me visualize the relationships.

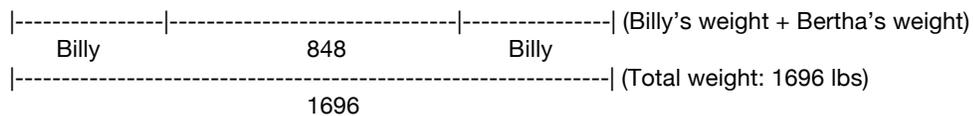
|-----| (Billy and Bertha combined: 1696 lbs)
|-----| (Difference between Billy and Bertha: 848 lbs, half of the total)

I wasn't sure how long the line should be for Billy, but that didn't matter because I just wanted to see the relationships.

|----| (Billy's weight: unknown)
|----|-----| (Bertha's weight: 848 + Billy's weight)
|----|-----|-----| (Total weight: Billy's weight + Bertha's weight)

Now I know that the total of the two, 1696 pounds, is made up of Billy's weight + 848 + Billy's weight

Billy's weight should be 1/4 of the total weight, like this:



Billy's weight should be $1696 / 4 = 424$ lbs. That means Bertha's weight is $424 + 848 = 1272$ lbs.

Check: $424 + 1272 = 1696$ lbs. That must be right!

Extra: Billy's weight is $1/4$ of their combined weights and Bertha's weight is $3/4$ of the combined weight. That means the proportion of Bertha's weight to Billy's weight is 3:1. The proportions for the butter cows would be 600:200 and the slices of toast would be 19200:6400 and the lifetimes would be $2:2/3$.

The butter cow Billy's size would cover 6400 slices of toast It would take $2/3$ of a lifetime to eat them all.

Method 5: Algebraic Reasoning

I know that I need to find Billy's weight. I know that

$$\text{Billy's weight} + 848 \text{ equals Bertha's weight}$$

I also know that

$$\text{Billy's weight} + \text{Bertha's weight equals } 1696 \text{ lbs.}$$

Using those two thoughts together, I know that

$$\text{Billy's weight} + \text{Billy's weight} + 848 \text{ lbs. equals } 1696 \text{ lbs.}$$

That means that

$$\text{Two of Billy's weight will equal } 848 \text{ lbs. and so Billy's weight would be } 424 \text{ lbs.}$$

If Billy's weight is 424 lbs. then Bertha weighs 1272 lbs.

Method 6: Algebra

If I let

$$x = \text{Billy's weight}$$

then I know that

$$x + 848 = \text{Bertha's weight}$$

I know that:

$$x + (x + 848) = \text{Billy's and Bertha's weight (total weight)}$$

but I also know

$$1696 = \text{total weight}$$

So, I can write this equation and solve for x:

$$x + (x + 848) = 1696$$

$$2x + 848 = 1696$$

$$2x = 848$$

$$x = 424$$

If Billy weighs 424 lbs., then Bertha weighs 1272 lbs.

Teaching Suggestions

As is the case in many of our problems, Kaytee is a real person, though not really a farmer. She's a photographer who, like Max, got to go to the Iowa State Fair in 2008. Max was looking for pictures of some awesome rides that might be included in a future GeoPoW when he saw those cows and immediately thought they could be the subject of a guess and check problem or a proportional reasoning problem. When Kaytee gave Max this photo she told him that her caption for it would be: "Rule of cuteness: any animal pictured with a smaller version of itself is automatically cute." We agree!

As we talked about this problem in the office we debated whether we should use the idea of a "guess the weight" contest since Bertha's and Billy's weights had constraints given in the problem. Some on staff thought it would be more accurate to say "calculate the weight" or "determine the weight." We decided, however, that using "guess the weight" might just hint to students the idea of using a guess

and check strategy. As you can see in the solution paths above and in the Activity Series document for this problem, guess and check provides opportunities for students to understand the relationships in the problem, organize their thinking carefully, and even notice some patterns that lead to algebraic thinking.

In general, there are many opportunities for students to use logical reasoning, drawings, or even beginner variables to help them understand the problem. Any time students are noticing things like, “hey, Bertha’s weight is made up of Billy’s weight + 848” or “the total weight is made up of Billy’s weight + 848 + Billy’s weight again” they are doing important algebraic thinking.

If students get stuck solving the problem, we often use guess and check language to help students diagnose where they are stuck. We ask students to diagnose if they can’t make a guess, if they don’t know what calculations they could do, or if they don’t know how they could check if their guess was right. This might help students who say, “I just don’t know what to do!”

In case you would like a copy of the problem without the question to either project for the class or to print for individuals or groups, we’ll be providing one for most of the problems this year, including this one. Just look for “Scenario Only” listed under the Teacher Support Materials for this problem.

The Problem Solving and Communication Activity Series document for this problem contains ideas and activities to help students experience get better at the Guess and Check strategy.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

Sample Student Solutions

focus on **Clarity**

In the solutions below, I’ve provided the scores the students would have received in the **Clarity** category of our scoring rubric. My comments focus on what I feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Explanation is very difficult to read and follow.	Another student might have trouble following the explanation. Long and written in one paragraph. Many spelling errors/typos.	Explains the steps that they <i>do</i> explain in such a way that another student would understand (needn’t be complete to be clear). Makes an effort to check formatting, spelling, and typing (a few errors are okay).	Format and organization make ideas exceptionally clear. Answer is very readable and appealing.

Garrett
age 12

Clarity
Novice

billy weighs 352 and berthas weighs 1344
 $1344+352=1696$

I notice that Garrett seems to have understood that Billy and Bertha’s weight needed to equal 1696 but I wonder what units Garrett was thinking would be attached to that number.

More importantly I’m wondering what is thinking was about the numbers he added.

Earl
age 11

Clarity
Novice

bertha weigth 1272 and billy weigth 424

I took 1696 and cut it in hafe and I got 848 and I cut that in hafe and I got 424 and I add 424 and 848 and it adds up to 1272 that's how much bertha Wight and billy weighs 424

I notice that Earl decided to halve 848 and then add it to itself. I wonder what prompted that thinking. I would encourage Earl to tell me more!

Sue
age 12

Clarity
Apprentice

Billy weighs 424 lbs. Bertha weighs 1,272 lbs.

1. The first thing I did was put down that Billy and Bertha = 1,696.
2. Then I wrote that Billy + 848 = Bertha.
3. After that I put down that 2 Billys + 848 = 1,696.
4. I then subtract 848 by 848 and 1,696 by 848.
5. $848 = 2$ Billy

I like how Sue has used steps to explain what she was thinking. I would encourage her to use units with her numbers and tell me a little bit of the "why" behind what she did.

Catherine
age 11

Clarity
Apprentice

Billy is 424 pounds. Bertha is 1,272 pounds which add up to 1,696 pounds all together.

To find Billy's and Bertha's weight I needed to make a chart of guess and check. I had to guess the pounds of Billy weight and then added 848 to see if it adds up to 1,696. If it didn't add up to 1,696 i needed to do the whole proses again.

I notice Catherine mentioned that she made a chart. I'd love to see her chart. I would encourage her to tell us how she came up with her first guess, what she thought about as she considered it and how she then decided her next guess.

Kristin
age 11

Clarity
Practitioner

Bertha's weight was 1272 pounds and Billy's weight was 424 pounds. Extra: Billy's butter cow would cover 6400 slices of toast and that would take a person 2/3 of a lifetime to consume.

As I read the problem two things jumped out at me, 1696 and 848 **more than** billy. So I decided to divide 1696 by 2.

$$1696 / 2 = 848$$

At first I thought that it was a trick question, because they can't be the same weight. Then I thought that can't be right so I looked at the problem again and thought "Why don't I divide 848 by 2?"

$$848 / 2 = 424$$

Then I added 424 to 848.

$$848 + 424 = 1272$$

After that I checked my work. The way I did that was I added 424 to 1272.

$$1272 + 424 = 1696$$

And that is how I got my answer which is bertha weighing 1272, and billy weighing 424.

Kristin has done a nice job explaining each of her steps in a way that another student could easily follow along. She's used line breaks, in particular for her equations.

I might encourage her to capitalize "Billy" and "Bertha" but that details doesn't detract from understanding her thinking.

Extra: when I looked at it I saw that I needed to use porportion. So I made a fraction out of Billy's and Bertha's weights next to x over Bertha's butter wieght. That was my first porportion.

$$\frac{424}{1272} = \frac{X}{600}$$

Then I cross-mutiplied.

$$\frac{424}{1272} = \frac{x}{600}$$

$$424 * 600 = 254400$$

$$1272 * X = 1272x$$

Next I divided.

$$\frac{254400}{1272} = \frac{1272x}{1272}$$

$$\frac{254400}{1272} = 200$$

$$X = 200$$

Then I did my next porportion using billy and bertha's butter weights and bertha's toast amount.

$$\frac{200}{600} = \frac{X}{19200}$$

I cross-mutiplied again.

$$\frac{200}{600} = \frac{X}{19200}$$

$$200 * 19200 = 3840000$$

Next I divided...Again.

$$\frac{3840000}{600} = \frac{600x}{600}$$

$$\frac{3840000}{600}$$

$$X = 6400$$

Then I divided twice.

$$19200 / 2 = 9600$$

$$6400 / 9600 = 2/3$$

And that is how I got my answer.

Neer
age 11

Clarity
Practitioner

Berta weighs 1,272 pounds and Billy 424 pounds. EXTRA- Billy's butter cow 6,400 slices of toast. Billy's slices of toast takes a person $\frac{2}{3}$ of a lifetime to eat.

I used a guess and check table to solve the question.

Bertha	1,096	1,396	1,296	1,270	1,269	1,271	1,272
Billy	600	300	400	426	427	425	424
Total	1,696	1,696	1,696	1,696	1,696	1,696	1,696
Difference	496	1,096	896	844	842	846	848

For each column I added Bertha and Billy and should have gotten 1,696 which I did. Then, I subtracted Bertha and Billy and got the difference which should be 848. As you can see I got 848 on the last one.

EXTRA- I did 1,272 divided 424 and got 3. Then, I wrote if you times Billy's weigh (424) by 3 you will get Bertha's weight (1,272). This means if you divide the weight of Billy's butter cow, the slices of toast, the number of by 3 you will get an answer.

This is what I did to get my answers.

I knew Bertha's butter cow covers 19,200 slices of toast so I did 19,200 divided by 3 and got 6,400. Then, I knew Billy's butter cow covers 6,400 slices of toast.

I knew Bertha's butter cow slices of toast takes a person two lifetimes to eat I did 2 divided by 2 and got $\frac{2}{3}$. Then, I knew Billy's butter cow slices of toast takes a person $\frac{2}{3}$ of a lifetime to eat.

Neer has used formatting to her advantage. Including a nicely formatted table makes it clear what she guessed. The color coding makes each category of numbers stand out. In addition she's used short paragraphs to make it easier to follow along with what she was thinking at each step. One suggestion I might make is that she include some labels to her chart.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Pre-Algebra Problems of the Week. Please let me know if you have ideas for making them more useful.

<https://www.nctm.org/contact-us/>