The contexts in *Investigating Fractions, Decimals, and Percents* are created through carefully-crafted posters. These images are also provided in a reproducible format in the appendix of each unit.

The concise outline of the day’s teaching moves is an ideal guide to reference as you teach.

**Materials Needed** lists all of the resources you and your students will use during the workshop.

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**Day One Outline**

**Developing the Context**

* Introduce the weird dials context and have students talk in pairs about what they notice about the dials and what they think the dials show.
* Convene a whole-group discussion of the dials, and record students’ observations on chart paper.
* Distribute Appendix C and invite students to investigate the dials.

**Supporting the Investigation**

* Students need to construct for themselves how the motions of the hands on the dials are related, so it is important not to tell them how to read the dials or explain to them about decimal notation.
* Encourage students to consider the differences in the numbers and remind them that each recording was done ten minutes apart.

**Preparing for the Math Congress**

* Ask students to make posters explaining what numbers they think the dials indicate and the rationale for their thinking.
* Plan for a congress discussion that will focus on the relationships among the dials.

**Facilitating the Math Congress**

* To encourage consideration of how the dials are related, scaffold a discussion that will culminate with students comparing parts of rotations on one dial to whole rotations of another dial.

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**Materials Needed**

- Weird dials poster, class version *(if you do not have the full-color poster (available from Heinemann), you can assemble a meter by making five copies of the dial on page 68)*

- Cut out the small hands and place a thumb tack through the center of each, enabling the hand to move on each dial.

- Set the hands to exactly match Appendix B before you begin.

- Weird dials poster, student version *(Appendix B)—one per student*

- Student recording sheets for the weird dials investigation *(Appendix C)—one per pair of students*

- Blank meters *(Appendix D)—several copies per pair of students*

- Large chart paper—one sheet per pair of students

- Large chart pad and easel *(or chalkboard or whiteboard)*

- Markers
The context for every unit’s investigation is carefully crafted to support the development of the big ideas, strategies, and models. It sets the stage for learning in a way that will intrigue children and ignite their imaginations.

A series of bulleted notes strategically placed in the side column help you navigate through each day. These highlight the key teaching moves you will want to attend to during each stage of the math workshop.

The main column contains step-by-step teaching advice, professional insights, and detailed suggestions for supporting and extending student learning.
The **Inside One Classroom** feature offers a glimpse into a classroom community of teachers and students as they explore the mathematics of the unit.

The **dialogue excerpts** model teaching language and are designed to help you envision interactions during minilessons, investigations, and math congresses.

The side column contains **notes**—my own professional insights on the dialogue and interaction.
The units in *Contexts for Learning* have been designed for maximum effectiveness. For example, the numbers are carefully chosen to represent landmark numbers or number relationships that are especially telling. **Behind the numbers**, explains the significance of the numbers chosen, why they are ordered the way they are, and how they work with the context to support the development of certain strategies, models, and big ideas.

**Supporting the Investigation**

In this investigation, students will grapple with the key ideas embedded in the context: that the rates at which hands turn are related by multiples of ten and that this understanding can be used to determine what the proper reading of the meter is. In effect, the meter is like one clock with five hands (except with separate dials for each hand and with the directions of rotation alternating). To read the meter, one records the highest digit completely passed by the hand on each dial (again being careful about the direction of rotation). But we do not directly teach this in this investigation because we first want students to construct the ideas that underlie the relationships of the dials. By constructing how the dials are related first, students will have a more robust understanding when they begin to work with decimals later in the unit.

Students may need to be reminded that each row of dials in Appendix C is Zig’s recording and that the recordings were made ten minutes apart. The time is important. The context of this investigation is structured so that students will see how the dials increase at regular time intervals and will therefore consider the relationships between the dials as they turn. As you move around and confer with students, encourage them to look at the differences and remind them that each recording was made ten minutes apart.
Recording sheets and other teaching tools are provided in a reproducible format at the end of the respective unit. They are also provided in an easy-to-access PDF format on the Teaching Resources CD-ROM.

Note how the questions on the recording sheet are formulated:
+ How do you think the hands on the dials move?
+ If you were to write down numbers for what you see, what would you write?

The questions have been formulated in this way because, at this point, the students should not be instructed on how to read the dials, nor is this the time to explain decimal notation. The objective here is for students to see that a hand on the right moves ten times faster than the hand on its immediate left and to use this idea to determine how to read the dials. In the days that follow, students will construct an understanding of how decimals work, but in order to make sense of them they need to first understand how the motions of the hands on the dials are related. As they record a sequence of numbers for what they believe the dials read, have them insert a period (decimal point) between the ones and the tenths numerals. This is social knowledge; students won't necessarily do this without your pointing out that the period is written between those two dials as part of the recording, and therefore it is a convention they should all follow. Have them use it because it is written there, but don't try to explain decimals. As you move around and confer, here are some strategies you can expect to see:

+ Recording the numbers on each dial as a way to determine the change—for example, realizing that in Zip's second reading the hand on the last dial (the thousandths) has moved 5 (from 1 to 6) and the hand on the next dial (the hundredths) has moved 2 (from 6 to 8).
+ Noticing the tick marks between the numbers and attempting to determine the relationship they have to the movement of the hands on the dials to the right. For example, a student might believe that when the first set of dials reads 35.901, the second dial in that sequence reads 5 \( \frac{9}{10} \),. The tick marks show the tenths of the unit of the dial. A reading of 35.900 could also be read as 3 \( \frac{9}{10} \) tens, 9 tenths, 0 hundredths, 0 thousandths (if a student believes that each dial is recorded separately and also looks at the tick marks and records them as well). As you confer with students, support them in examining the relationships among the dials. They will learn to record one number for the meter shortly.
+ Deciding to write the numbers in a column, like this:
  
  | 35.961 |
  | 35.986 |
  | 36.011 |
  | 36.026 |
  | 36.061 |

Students might then notice the sequence of the digits appearing vertically, and attempt to continue the sequence in order to examine what happens with the hands on the second and third dials—when will the hand pointing to 0 on the third dial point to 1 and when will the hand pointing to the 6 on the second dial point to 7? Provide students with pictures of blank meters (Appendix D) so they can continue the sequence if they wish.
Conferring with Students at Work

Maria: The first row of digits is 3 6 0 6 1.

Lucy: I'm not sure. Why is the third number a 0 and not a 9? It's in between the two numbers.

Maria: Yeah, but then we don't know what any of them are. If we don't call it zero, what do we call it? What do we do? Can you help us?

John (the teacher): Maybe you should talk about how you think the hands move.

Maria: You mean like if they were a clock?

John: That's a good start.

Lucy: But in a clock there are two hands but the hour hand goes a lot slower than the numbers.

John: Think about that. Are the hands ever in between numbers? I'll check back with you a bit later.

(John moves on to confer with another pair of students.)

Alain: (Addressing John) We're talking about all the little lines between the numbers. They must do something.

Toni: Like they're there to confuse us.

Alain: No, really. I think there are two numbers on each clock. They just didn't put all the lines on.

Toni: Oh, I get it, the first row of clocks ends, let's see, 35, 59, 96, 61, and 10. That's a lot of time zones.

John: It seems that some of the digits are repeating.

Alain: Yeah. The last number is the first.

John: That's interesting. Where was the hand pointing on the second dial when it was 9 on the third dial?

Alain: It was almost on the 6... like on the little ninth mark... 9 out of 10 of the little marks.

John: And then when the third dial went to 0 and the second one went to 0? (Referring to the third row, 36017)

Toni: Oh... now it is on the 6... just about.
Preparing for the Math Congress offers strategies for helping your students organize and present their findings and tips on how to orchestrate and scaffold powerful discussions.
The math congress continues the work of helping children become mathematicians in a mathematics community—it is a forum in which children communicate their ideas, solutions, problems, proofs, and conjectures to one another. Out of the congress come ideas and strategies that form the emerging discipline of mathematics in the classroom.

### A Portion of the Math Congress

**Author’s Notes**

During the congress, John initiated the discussion not only on what any two given dial reads, but also on how the dials relate to each other. Many of the students may still see the dials as separate entities, not as parts of a single readout. It is to be expected at this point that different groups will have conflicting readings. John uses these as a way to encourage discussion of the relationships. Conflicting readings can generate disequilibrium and provide for a rich discussion.

**John:** The teacher: Edgar and Rhonda, tell us what you think the first dial reads.

**Edgar:** Well, I thought the first one read 36.06, but Rhonda disagreed. She thought it read 35.96.

**Rhonda:** No, on this third dial, it looks like it’s on the 0, but it’s not quite there yet. I was thinking that they work like clocks, so since it’s not quite on the 0 yet, it should count as a 3.

**Edgar:** I still think it’s a 0, though.

**John:** Rhonda, can you maybe explain more?

**Rhonda:** Like on a clock, if it was 2:10, then the hour hand would be between the 2 and the 3. Since it’s not quite on the 3 yet, the hour is 2.

**Edgar:** But there’s no minute hand.

**John:** Hmm, that is confusing, isn’t it? It seems we have some questions about what to do here. Sasha, you and Carmen started off recording in a different way. Would you share next? Tell us what you did.

**Sasha:** We saw that the dials had smaller marks between the numbers, so we looked at where the hand was pointing. In the first dial, it’s past the 2 and on the fifth mark, so we recorded the first dial as 15.

**Carmen:** We kept doing that and got 35, 59, 96, 61, 10.

*continued on next page*
Underscoring our ongoing and authentic approach to evaluation, regular assessment tips are provided where and when you need them.

Each day ends with reflections on the big ideas and strategies students explored during the workshop.
Appendix C  Student recording sheet for the weird dials investigation

Names _______________________ Date ________________

Zig’s notes:

I noticed that the hands move. I will watch really carefully and every ten minutes I will draw a picture and record what the dials show.

Here are my results. Wow! I think I am beginning to see how the dials are related.

What do you think Zig means? How do you think the hands on the dials move? If you were to write down numbers for what you see, what would you write?

Teaching tools like this student recording sheet appear in a reproducible format in the back of each unit. They are also provided in an easy-to-access PDF format on the Teaching Resources CD-ROM.

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