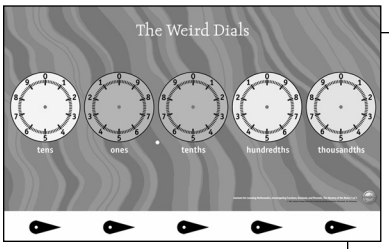


The contexts in *Investigating Fractions, Decimals, and Percents* are created through carefully-crafted **posters.** These images are also provided in a reproducible format in the appendix of each unit.

The **concise outline of the day's teaching moves** is an ideal guide to reference as you teach.

Materials Needed lists all of the resources you and your students will use during the workshop.

DAY ONE

The Weird Dials

Today you will tell the story of a boy named Zig who finds five weird dials on the side of his house. These dials form the context for the investigations in the unit. Although he does not yet know that the dials are part of an electric meter, Zig investigates their mathematical properties. Like Zig, the students in the class study the relationships between the dials as their hands turn. These initial inquiries will lay the foundation for the big ideas regarding equivalence, place value, and powers of ten that the students need for later work with decimals.

Day One Outline

Developing the Context

- Introduce the weird dials context and have students talk in pairs about what they notice about the dials and what they think the dials show.
- Convene a whole-group discussion of the dials, and record students' observations on chart paper.
- Distribute Appendix C and invite students to investigate the dials.

Supporting the Investigation

- Students need to construct for themselves how the motions of the hands on the dials are related, so it is important not to tell them how to read the dials or explain to them about decimal notation.
- Encourage students to consider the differences in the numbers and remind them that each recording was done ten minutes apart.

Preparing for the Math Congress

- Ask students to make posters explaining what numbers they think the dials indicate and the rationale for their thinking.
- Plan for a congress discussion that will focus on the relationships among the dials.

Facilitating the Math Congress

- To encourage consideration of how the dials are related, scaffold a discussion that will culminate with students comparing parts of rotations on one dial to whole rotations of another dial.

Materials Needed

Weird dials poster, class version [If you do not have the full-color poster (available from Heinemann), you can assemble a meter by making five copies of the dial on page 68.]

Cut out the small hands and place a thumbtack through the center of each, enabling the hand to move on each dial. Set the hands to exactly match Appendix B before you begin.

Weird dials poster, student version (Appendix B)—one per student

Student recording sheets for the weird dials investigation (Appendix C)—one per pair of students

Blank meters (Appendix D)—several copies per pair of students

Large chart paper—one sheet per pair of students

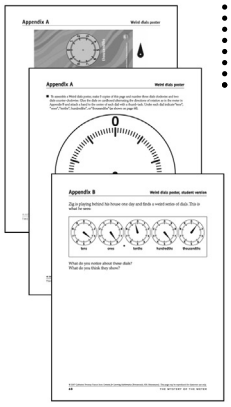
Large chart pad and easel (or chalkboard or whiteboard)

Markers

Day One 13

Developing the Context

- Introduce the weird dials context and have students talk in pairs about what they notice about the dials and what they think the dials show.
- Convene a whole-group discussion of the dials, and record students' observations on chart paper.
- Distribute Appendix C and invite students to investigate the dials.



Begin this unit by telling a story about a boy named Zig who has found a weird set of dials on the side of his house and wonders what they are. Later Zig will find out from his parents what the dials do, but initially he can only explore them using ingenuity and careful observation.

The unit begins with students in the same position as Zig. They don't know what these dials are or how they work—they are a mystery. Students are simply shown the weird dials poster and asked to investigate—to record what they notice. Provide students with copies of the student version of the poster (Appendix B) and ask them to discuss with the person sitting next to them the following two questions:

- *What do you notice about these dials?*
- *What do you think they show?*

Pose these questions carefully and allow ample time for discussion. It is critical to allow students time to make their own observations about the meter and to construct their own language about what they think the dials show. Do *not* explain that the dials are a meter or how it works. This understanding will emerge in time as students reconstruct ideas about place value in the context of the meter and think about decimals. If a student recognizes the dials as those of an electric meter, you should note that that is one possibility, but suggest that maybe they are something else instead. In fact, the meter is slightly different from the usual electric meter on most homes and exactly how will emerge later.

After a few minutes of pair talk, have students share their observations about the dials and record them on chart paper. This list will be important over the next few days. Each of the observations becomes a mystery to be explained in subsequent days. It is also an opportunity for the class to develop language that they will need to explain their ideas about how decimal place value works. Common observations include the following: there are five "clocks"; the numbers on the "clocks" go only to nine; each "clock" is missing a hand; some of the "clocks" go backward; there are little marks between the numbers; the "clocks" are like the clocks for different time zones at an airport. *[See Figure 1]*

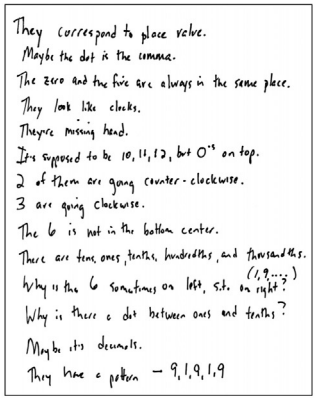


Figure 1

The **context** for every unit's investigation is carefully crafted to support the development of the big ideas, strategies, and models. It sets the stage for learning in a way that will intrigue children and ignite their imaginations.

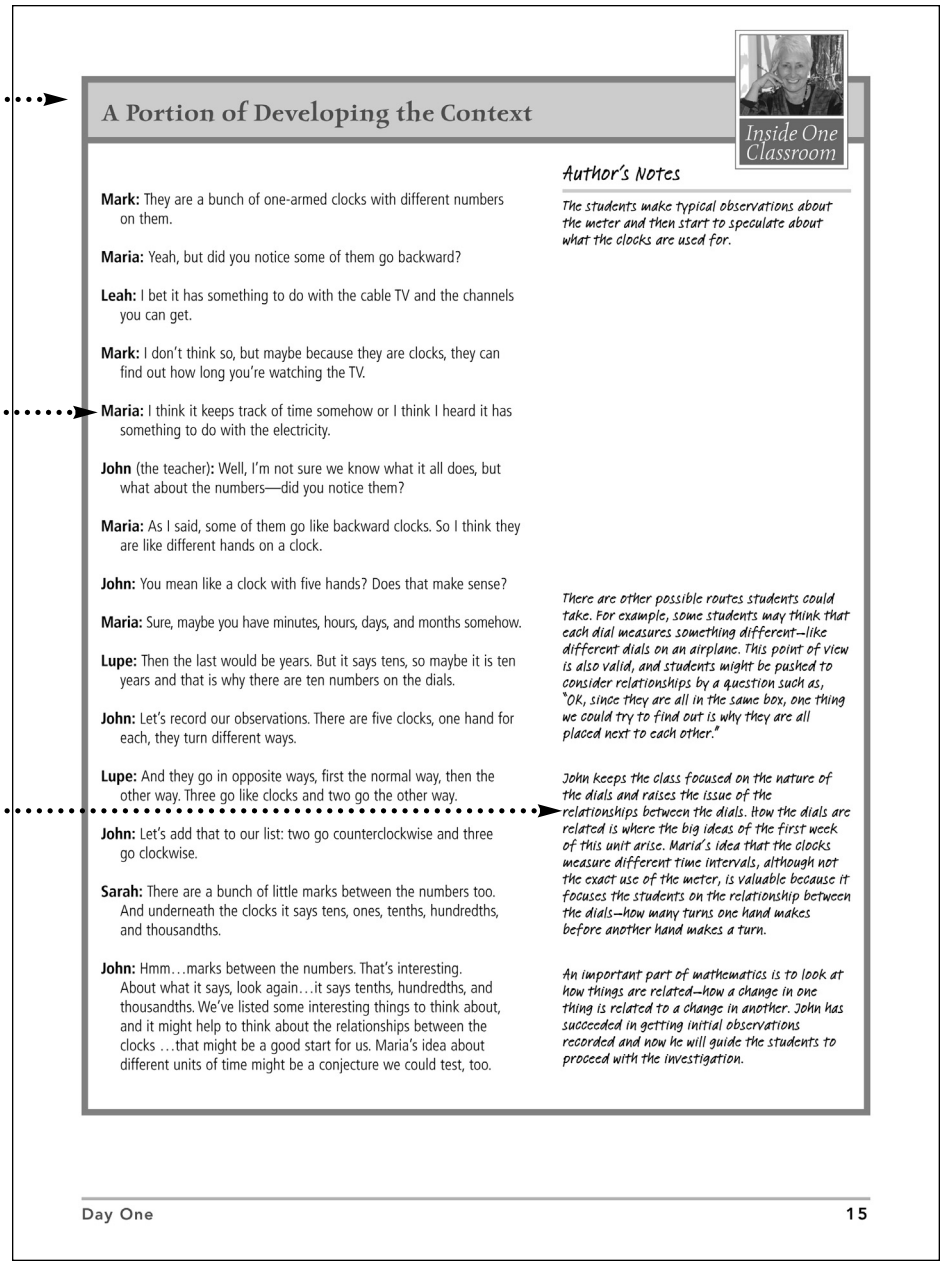
A series of bulleted notes strategically placed in the side column help you navigate through each day. These highlight the key **teaching moves** you will want to attend to during each stage of the math workshop.

The main column contains **step-by-step teaching advice,** professional insights, and detailed suggestions for supporting and extending student learning.

The **Inside One Classroom** feature offers a glimpse into a classroom community of teachers and students as they explore the mathematics of the unit.

The **dialogue excerpts** model teaching language and are designed to help you envision interactions during minilessons, investigations, and math congresses.

The side column contains **notes**—my own professional insights on the dialogue and interaction.



A Portion of Developing the Context

Mark: They are a bunch of one-armed clocks with different numbers on them.

Maria: Yeah, but did you notice some of them go backward?

Leah: I bet it has something to do with the cable TV and the channels you can get.

Mark: I don't think so, but maybe because they are clocks, they can find out how long you're watching the TV.

Maria: I think it keeps track of time somehow or I think I heard it has something to do with the electricity.

John (the teacher)**:** Well, I'm not sure we know what it all does, but what about the numbers—did you notice them?

Maria: As I said, some of them go like backward clocks. So I think they are like different hands on a clock.

John: You mean like a clock with five hands? Does that make sense?

Maria: Sure, maybe you have minutes, hours, days, and months somehow.

Lupe: Then the last would be years. But it says tens, so maybe it is ten years and that is why there are ten numbers on the dials.

John: Let's record our observations. There are five clocks, one hand for each, they turn different ways.

Lupe: And they go in opposite ways, first the normal way, then the other way. Three go like clocks and two go the other way.

John: Let's add that to our list: two go counterclockwise and three go clockwise.

Sarah: There are a bunch of little marks between the numbers too. And underneath the clocks it says tens, ones, tenths, hundredths, and thousandths.

John: Hmm…marks between the numbers. That's interesting. About what it says, look again…it says tenths, hundredths, and thousandths. We've listed some interesting things to think about, and it might help to think about the relationships between the clocks …that might be a good start for us. Maria's idea about different units of time might be a conjecture we could test, too.

Author's Notes

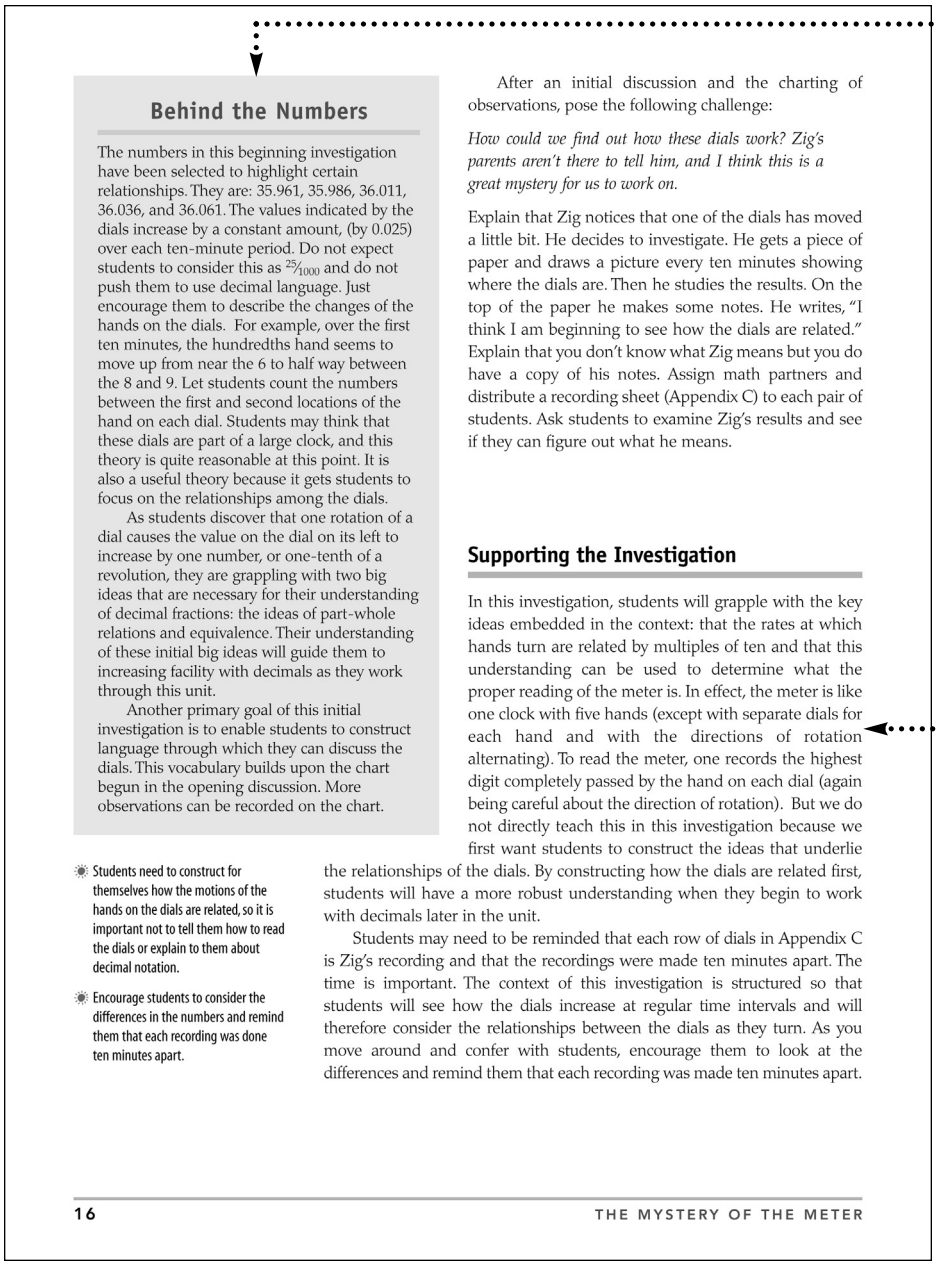
The students make typical observations about the meter and then start to speculate about what the clocks are used for.

There are other possible routes students could take. For example, some students may think that each dial measures something different—like different dials on an airplane. This point of view is also valid, and students might be pushed to consider relationships by a question such as, "OK, since they are all in the same box, one thing we could try to find out is why they are all placed next to each other."

John keeps the class focused on the nature of the dials and raises the issue of the relationships between the dials. How the dials are related is where the big ideas of the first week of this unit arise. Maria's idea that the clocks measure different time intervals, although not the exact use of the meter, is valuable because it focuses the students on the relationship between the dials—how many turns one hand makes before another hand makes a turn.

An important part of mathematics is to look at how things are related—how a change in one thing is related to a change in another. John has succeeded in getting initial observations recorded and now he will guide the students to proceed with the investigation.

Day One 15



Behind the Numbers

The numbers in this beginning investigation have been selected to highlight certain relationships. They are: 35.961, 35.986, 36.011, 36.036, and 36.061. The values indicated by the dials increase by a constant amount, (by 0.025) over each ten-minute period. Do not expect students to consider this as $^{25}/_{1000}$ and do not push them to use decimal language. Just encourage them to describe the changes of the hands on the dials. For example, over the first ten minutes, the hundredths hand seems to move up from near the 6 to half way between the 8 and 9. Let students count the numbers between the first and second locations of the hand on each dial. Students may think that these dials are part of a large clock, and this theory is quite reasonable at this point. It is also a useful theory because it gets students to focus on the relationships among the dials.

As students discover that one rotation of a dial causes the value on the dial on its left to increase by one number, or one-tenth of a revolution, they are grappling with two big ideas that are necessary for their understanding of decimal fractions: the ideas of part-whole relations and equivalence. Their understanding of these initial big ideas will guide them to increasing facility with decimals as they work through this unit.

Another primary goal of this initial investigation is to enable students to construct language through which they can discuss the dials. This vocabulary builds upon the chart begun in the opening discussion. More observations can be recorded on the chart.

After an initial discussion and the charting of observations, pose the following challenge:

How could we find out how these dials work? Zig's parents aren't there to tell him, and I think this is a great mystery for us to work on.

Explain that Zig notices that one of the dials has moved a little bit. He decides to investigate. He gets a piece of paper and draws a picture every ten minutes showing where the dials are. Then he studies the results. On the top of the paper he makes some notes. He writes, "I think I am beginning to see how the dials are related." Explain that you don't know what Zig means but you do have a copy of his notes. Assign math partners and distribute a recording sheet (Appendix C) to each pair of students. Ask students to examine Zig's results and see if they can figure out what he means.

Supporting the Investigation

In this investigation, students will grapple with the key ideas embedded in the context: that the rates at which hands turn are related by multiples of ten and that this understanding can be used to determine what the proper reading of the meter is. In effect, the meter is like one clock with five hands (except with separate dials for each hand and with the directions of rotation alternating). To read the meter, one records the highest digit completely passed by the hand on each dial (again being careful about the direction of rotation). But we do not directly teach this in this investigation because we first want students to construct the ideas that underlie the relationships of the dials. By constructing how the dials are related first, students will have a more robust understanding when they begin to work with decimals later in the unit.

Students may need to be reminded that each row of dials in Appendix C is Zig's recording and that the recordings were made ten minutes apart. The time is important. The context of this investigation is structured so that students will see how the dials increase at regular time intervals and will therefore consider the relationships between the dials as they turn. As you move around and confer with students, encourage them to look at the differences and remind them that each recording was made ten minutes apart.

- Students need to construct for themselves how the motions of the hands on the dials are related, so it is important not to tell them how to read the dials or explain to them about decimal notation.
- Encourage students to consider the differences in the numbers and remind them that each recording was done ten minutes apart.

The units in *Contexts for Learning* have been designed for maximum effectiveness. For example, the numbers are carefully chosen to represent landmark numbers or number relationships that are especially telling. **Behind the numbers,** explains the significance of the numbers chosen, why they are ordered the way they are, and how they work with the context to support the development of certain strategies, models, and big ideas.

The **heart of math workshop** consists of ongoing investigations developed within contexts and situations that enable children to mathematize their lives. As children work, the teacher moves around the classroom, listening, conferring, supporting, challenging, and celebrating.

Recording sheets and other teaching tools are provided in a reproducible format at the end of the respective unit. *They are also provided in an easy-to-access PDF format on the* Teaching Resources CD-ROM.

Note how the questions on the recording sheet are formulated:

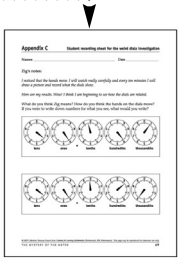
- *How do you think the hands on the dials move?*
- *If you were to write down numbers for what you see, what would you write?*

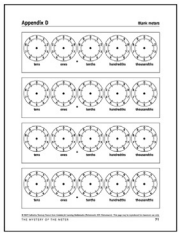
The questions have been formulated in this way because at this point the students should not be instructed on how to read the dials, nor is this the time to explain decimal notation. The objective here is for students to see that a hand on the right moves ten times faster than the hand on its immediate left and to use this idea to determine how to read the dials. In the days that follow students will construct an understanding of how decimals work, but in order to make sense of them they need to first understand how the motions of the hands on the dials are related. As they record a sequence of numbers for what they believe the dials read, have them insert a period (decimal point) between the ones and the tenths numerals. This is *social knowledge;* students won't necessarily do this without your pointing out that the period is written between those two dials as part of the recording, and therefore it is a convention they should all follow. Have them use it because it is written there, but don't try to explain decimals! As you move around and confer, here are some strategies you can expect to see:

- Recording the numbers on each dial as a way to determine the change—for example, realizing that in Zig's second reading the hand on the last dial (the thousandths) has moved 5 (from 1 to 6) and the hand on the next dial (the hundredths) has moved 2 (from 6 to 8).
- Noticing the tick marks between the numbers and attempting to determine the relationship they have to the movement of the hands on the dials to the right. For example, a student might believe that when the first set of dials reads 35.961, the second dial in that sequence reads 5 $\frac{9}{10}$+. The tick marks show the tenths of the unit of the dial. A reading of 35.900 could also be read as 3 $\frac{5}{10}$ tens, 5 $\frac{9}{10}$ ones, 9 tenths, 0 hundredths, 0 thousandths (if a student believes that each dial is recorded separately and also looks at the tick marks and records them as well). As you confer with students, support them in examining the relationships among the dials. They will learn to record one number for the meter shortly.
- Deciding to write the numbers in a column, like this:

35.961
35.986
36.011
36.036
36.061

Students might then notice the sequence of the digits appearing vertically, and attempt to continue the sequence in order to examine what happens with the hands on the second and third dials—when will the hand pointing to 0 on the third dial point to 1 and when will the hand pointing to the 6 on the second dial point to 7? Provide students with pictures of blank meters (Appendix D) so they can continue the sequence if they wish.


Appendix C Student recording sheet for the wind data investigation


Appendix D Blank meters

Conferring with Students at Work

Maria: The first row of clocks is 3 6 0 6 1.

Lucy: I'm not sure. Why is the third number a 0 and not a 9? It's in between the two numbers.

Maria: Yeah, but then we don't know what any of them are. If we don't call it zero, what do we call it? What do we do? Can you help us?

John (the teacher)**:** Maybe you should talk about how you think the hands move.

Maria: You mean like if they were a clock?

John: That's a good start.

Lucy: But in a clock there are two hands but the hour hand goes a lot slower between the numbers.

John: Think about that. Are the hands ever in between numbers? I'll check back with you a bit later.

(John moves on to confer with another pair of students.)

Alain: *(Addressing John.)* We're talking about all the little lines between the numbers. They must do something.

Toni: Like they're there to confuse us.

Alain: No, really, I think there are two numbers on each clock. They just didn't put all the lines on.

Toni: Oh, I get it, the first row of clocks reads, let's see, 35, 59, 96, 61, and 10. That's a lot of time zones.

John: It seems that some of the digits are repeating.

Alain: Yeah. The last number is the first.

John: That's interesting. Where was the hand pointing on the second dial when it was 9 on the third dial?

Alain: It was almost on the 6…like on the little ninth mark…9 out of 10 of the little marks.

John: And then when the third dial went to 0 and the second one went to 6? *(Referring to the third row, 36.011.)*

Toni: Oh…now it is right on the 6…just about.

continued on next page

Author's Notes

As John confers, he pays attention to whether students read one dial at a time or if they examine them collectively across the meter. Maria and Lucy are initially considering each dial separately. John encourages them to think about how the clocks are related and about the positions between the numbers.

Sometimes the best way to confer is to support students to determine a way to start and then to let them explore on their own. John suggests thinking about the movement of the hands like on a clock and then he leaves the students to continue reflecting. Implicitly his action says, "I trust you to work on this and to figure it out." If John had stayed there, it is quite likely the students would have kept relying on him, requesting that he answer their questions.

John uses a question to encourage continuing reflection on the relationship among the dials.

Preparing for the Math Congress offers strategies for helping your students organize and present their findings and tips on how to orchestrate and scaffold powerful discussions.

continued from previous page

John: Hmmm…that's interesting, isn't it? If you were Zig, do you think you could figure out what the dials might say if you kept looking for more ten-minute intervals? I wonder when the 6 will almost be on the 7? What would the other dials have to read to make that happen?

By encouraging Toni and Alain to continue the pattern of the change every ten minutes, John focuses them directly on the relationship among the dials.

The numbers indicated on the dials increase by equal amounts every ten minutes. As students record they may not realize this, especially because of the ambiguity of what to record when dial hands are between numbers. Many students will select the number closest to the marker. Listen for conversations about this choice—these may be good clues for groups to share in the math congress. If students always choose the closest number, don't press them to change during the inquiry; instead, make a note to be sure they raise this issue during the congress. Sorting this out is crucial for their development.

Preparing for the Math Congress

Distribute a sheet of large chart paper to each pair of students and ask them to prepare a poster listing the numbers that they believe the dials indicate and explaining why they read the meter in this way. Rather than having students draw figures of meters repeatedly, provide students with multiple copies of the blank meters (Appendix D) so they can concentrate on placing hands on the dials. As students work on their posters, think about how you will structure the congress.

- Ask students to make posters explaining what numbers they think the dials indicate and the rationale for their thinking.
- Plan for a congress discussion that will focus on the relationships among the dials.

Tips for Structuring the Math Congress

It is helpful to choose two or three pairs of students to share during the congress. Ask them to share their work one at a time and use the work as the focus of discussion. Keep in mind that the purpose of this math congress is for students to have a conversation about the movements of the hands on the dials—the relationships between the dials. Some students may have correct numerical recordings but may not be able to explain why they chose the lower number when the arrow was between numbers. Such a pair might be good to start with because you can prompt conversation about the "why" with the whole class. Those that disagree should be encouraged to speak up. Some groups may forget the rotation reversal and they can be heard as well. Another good choice as a way to begin is to compare two different readings for the same set of dials. This comparison may evoke a discussion of the tick marks and the equivalence, for example, $35\frac{9}{10} = 35.9$. To deepen this understanding, next choose a pair whose work would initiate a discussion of the relationship between rotations of one dial, and how much the dial to the left of it rotates. This topic, with students comparing parts of rotations on one dial to whole rotations of another dial and why they are equivalent, is what the students should be discussing toward the end of the congress.

Facilitating the Math Congress

To encourage consideration of how the dials are related, scaffold a discussion that will culminate with students comparing parts of rotations on one dial to whole rotations of another dial.

Choose several posters reflecting different approaches to reading the dials and explain that since the use of the dials is a mystery, the important discussion is about *how they are related and how they change.* As students present their theories, they will need to justify their reasoning. An important issue will be whether students record the number closest to where the hand is pointing or if they round up or down.

Some students will record the dial readings by writing down the digit closest to the pointer. This issue should come up in the congress. The notion that the dials are some sort of a clock is useful, and during the investigation students might ask if this is what happens on a clock. How does the position of the hand determine the hour? This way they can consider the question of the relationship between the rotation of the hands on the dials and the tick marks.

A Portion of the Math Congress

John (the teacher)**:** Edgar and Rhonda, tell us what you think the first dial reads.

Edgar: Well, I thought the first one read 36.061, but Rhonda disagreed. She thought it read 35.961.

Rhonda: See, on this third dial, it looks like it's on the 0, but it's not quite there yet. I was thinking that they work like clocks, so since it's not quite on the 0 yet, it should count as a 9.

Edgar: I still think it's a 0, though.

John: Rhonda, can you maybe explain more?

Rhonda: Like on a clock, if it was 2:30, then the hour hand would be between the 2 and the 3. Since it's not quite on the 3 yet, the hour is 2.

Edgar: But there's no minute hand.

John: Hmm, that is confusing, isn't it? It seems we have some questions about what to do here. Sasha, you and Carmen started off recording in a different way. Would you share next? Tell us what you did.

Sasha: We saw that the dials had smaller marks between the numbers, so we looked at where the hand was pointing. In the first dial, it's past the 3 and on the fifth mark, so we recorded the first dial as 35.

Carmen: We kept doing that and got 35, 59, 96, 61, 10.

continued on next page

Author's Notes

During the congress, John focuses the discussion not only on what any one given dial reads, but also on how the dials relate to each other. Many of the students may still be seeing the dials as separate entities, not as part of a single reading.

It's to be expected at this point that different groups will have conflicting readings. John uses these as a way to encourage discussion of the relationships. Conflicting readings can engender disequilibrium and provide for a rich discussion.

John acknowledges the confusion and asks Sasha and Carmen to share next. Their readings are quite different. Comparing conflicting readings encourages students to examine relationships.

The math congress continues the work of helping children become mathematicians in a mathematics community—it is a forum in which children communicate their ideas, solutions, problems, proofs, and conjectures to one another. Out of the congress come ideas and strategies that form the emerging discipline of mathematics in the classroom.

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John: Did you notice any pattern in your numbers?

By asking about patterns, John reinforces that mathematicians look for patterns. They investigate them and wonder why they are occurring.

Sasha: It was weird. The second digit in one number was the first digit in the next.

Carmen: That kept happening, but we couldn't figure out why.

Sam: The name under the first dial says tens. Maybe the little black marks are like for fractions or something. I think Rhonda is right. It should be 3. I think it is like $3\frac{1}{2}$ or something like that.

Focusing on the relationships with amounts they understand, like tens, helps students begin to see possible connections between the dials.

John: Turn to the person next to you and talk about what Sam said. What are your thoughts about these little marks? *(Allows time for pair talk.)* Sasha?

Pair talk heightens reflection and implicitly says this is an important idea for all to consider as a community.

Sasha: Yeah, I think Sam is right. So $3\frac{1}{2}$ tens would be 35!

Equivalence is now at the heart of the discussion.

Rhonda: I agree, too. See…it is like a clock. You wouldn't say four o'clock, you would say three and some minutes. I think the little marks are sort of like minutes.

John: So let's look at Carmen and Sasha's chart again. You have 59 for the second dial. What are you thinking now?

Carmen: Maybe it is 5 and $\frac{9}{10}$? Maybe that is why it's also 5.9, like Rhonda said. I don't know…I'm not sure. But I don't think we should write a 6. I disagree with Edgar.

Allowing students to disagree with each other is valuable as it pushes them to develop reasons for their assertions.

Underscoring our ongoing and authentic approach to evaluation, regular **assessment tips** are provided where and when you need them.

Assessment Tips

It is easy in mathematics for students to figure out a useful rule and apply it without understanding the deep connections behind the rule. Watch for students who may have reduced the day's investigation to the rule that you always choose the lower number in reading a dial, and ensure that they are thinking about the motion of the hand on the dial to the right of the one being read, and how that right-hand dial requires a full revolution to move the dial on its left one-tenth of a revolution.

Each day ends with **reflections** on the big ideas and strategies students explored during the workshop.

Reflections on the Day

Today students investigated five mysterious dials and how they changed after ten-minute intervals. The important issue that students are considering is the relationship between the dials as they move. These relationships form the basis for the development of two big ideas: (1) equivalence, and (2) how the wholes shift in decimal representations under multiplication and division by tens. These ideas underlie the concept of place value needed to understand decimals. As the unit progresses, these ideas will be explored more deeply.

Appendix C — Student recording sheet for the weird dials investigation

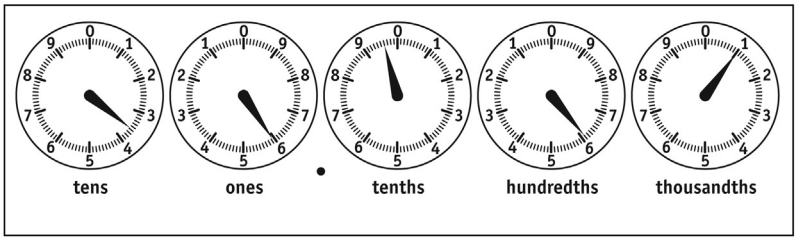
Names _____ Date _____

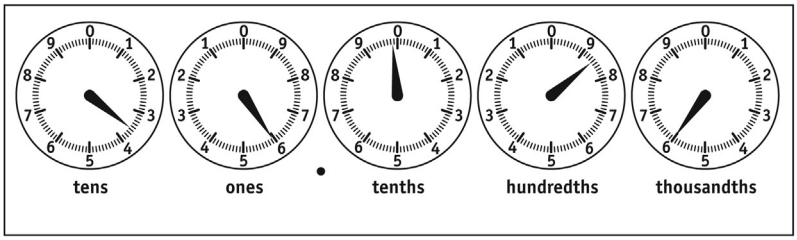
Zig's notes:

I noticed that the hands move. I will watch really carefully and every ten minutes I will draw a picture and record what the dials show.

Here are my results. Wow! I think I am beginning to see how the dials are related.

What do you think Zig means? How do you think the hands on the dials move? If you were to write down numbers for what you see, what would you write?





Teaching tools like this student recording sheet appear in a reproducible format in the back of each unit. *They are also provided in an easy-to-access PDF format on the* Teaching Resources CD-ROM.