Comprehending Math

Adapting Reading Strategies to Teach Mathematics, K–6

ARTHUR HYDE

Foreword by Ellin Oliver Keene

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Sometimes, in schools and classrooms, we simply confuse the help. By help I mean those pesky kids that show up every day expecting to be taught. In our best effort to teach the curriculum, use best instructional practices, create warm, inviting spaces in which kids can learn, prepare them for the tests, respond to individual needs, engage kids in the excitement of learning, integrate around themes, respond to parents, weave in resources from the community, get excited about our principal’s latest project, share and plan with our colleagues, read the latest in educational research and practice, go to the best conferences, write thoughtful report card comments, sit on the hospitality committee, and (whew) enjoy children—we just end up confusing the help.

We are trying to apply all we know about teaching and learning and, instead, find ourselves running around the classroom trying to do a little bit of this and a little bit of that. Eventually, we realize through our utter exhaustion that the kids have been watching us fly around like they’re enjoying a great tennis match, heads swaying as they follow us (or try to). They must be thinking, “Wow, look at her go!” We are trying to do so much, we are trying to do it well, we are trying to keep them engaged, and, in the end, we are probably confusing the help and it is we who end up doing the heavy lifting—cognitively speaking. I’ve always wondered: If we understand the basic principles that underlie human learning—the ability to retain and reapply concepts in new contexts—why is everything so difficult, so different, from class to class, discipline to discipline?

I had something close to an epiphany about all this while watching a lesson in a first/second-grade classroom years ago. The brilliant teacher, my friend and colleague, Colleen Buddy, was teaching inference, and on this particular day was trying to help the children understand that predictions are one form of inference. A young man raised his hand with a simple inquiry. “Why, Mrs. Buddy,” he said, tongue thrusting wildly through the space where front teeth are usually found, “why do you call it predicting when we’re talking about reading, hypothesizing [which
came out as hypothethitthing] when we’re in science, and estimating [ethimathing] when we’re in math? Aren’t they really all the same thing?”

We might just be confusing the help. Our young friend was merely saying, “Teachers, please! If we’re talking about the same kinds of thinking, though we are in different subject areas (your inauthentic scheduling needs, not ours), might we do the common-sense thing and refer to thinking into the future, for example, as the same process, no matter what time in the day we happen to discuss it?” Pick one term, they seem to be saying—hypothesizing, estimating, predicting—pick one and stick with it—it will make so much more sense to us.

Well! Of all the outrageous suggestions! But, wait a minute. Why do we compartmentalize learning and thinking throughout the day? Why does this segmentation increase as students get older? Why do we refer to one set of thinking processes during reading and another altogether during math? In the words of our precocious first grader, “Aren’t they really all the same thing?” Perhaps if we’re in the math or cognition departments at a major university, we may quibble a bit with our first grader’s notion. Perhaps there are subtle and important differences between estimating and predicting when one is working at the highest levels of theoretical research. But for our purposes in kindergarten through twelfth-grade classrooms, might we confuse the help less if we used the same language to describe similar thinking processes throughout the learning day and around the school?

Arthur Hyde has also had this epiphany, and the reader of this important book will be the richer for it. Hyde has woven together—no, braided—the concepts of thinking, language, and math, and has made a crystal-clear case for the application to math of the language and learning processes many teachers incorporate routinely into their literacy instruction. Why, he reasons, if we ask kids to construct meaning in reading, wouldn’t we ask them to do the same in math? Why not create a mathematically literate environment in the same way we strive to create literate environments for children learning to read, write, speak, and listen? How, he asks, can we fail to incorporate such a critical concept as revision into children’s mathematical lives? Revision is indelibly woven into our language and literacy classrooms, but do we, in Hyde’s words, forgive math mistakes, encouraging children to revise their mathematical thinking?

When reading this superb volume, you will be tempted, numerous times, I’m afraid, to strike your forehead with the heel of your hand and say, “Oh, why on earth didn’t I think of that?” The very concepts that have led to a revolution in reading comprehension instruction are here applied, with tremendous clarity, to the teaching of mathematics. Hyde relies on his rock-solid understanding of human cognition in order to relate thinking and language to the process of learning mathematics, a connection a certain first grader made years ago, but one which we educators have been slow to grasp.
This book reminded me of my worry that we elementary teachers may have had less than stellar experiences in learning mathematics in our own K–12 experience. As a result, we tend to draw conclusions about our efficacy as teachers of math and to make some very subtle, perhaps subconscious, decisions about our students’ propensity (lack thereof) to learn and to love the science of patterns—mathematics. Hyde provides a way through those obstacles for teachers like me who were less than excited about math. He helps us understand critical concepts in mathematics by way of our well-developed knowledge of comprehension strategies. Now, there’s a novel idea—how about helping us learn new ways of approaching math instruction by resting new concepts on our existing knowledge?!?! Is that a text-to-self connection, by any chance? Might someone have had the common sense to apply what we know about student learning to professional learning? Read on and be amazed!

In each of Hyde’s chapters on major comprehension (thinking) strategies, he lays out, with great good humor and insight-provoking examples, the thinking and language applications to mathematics in our classrooms. He makes the case, not only for using the language of the literacy classroom throughout the day, but to apply well-proven research concepts such as schema theory and metacognition to the fundamentally important problem-solving processes on which mathematical understanding rests.

As I was reading, I remembered an eighth grader named Tony who approached me at a middle school in Bridgeport, Connecticut, several years ago and asked if I was aware that there was a conspiracy going on at that school. Amused, but trying to keep a straight face, I asked him to tell me about the conspiracy. He glanced around him, checking for listening devices, I suppose, and said, “I just think you should know that the teachers around here are all teaching the same thing at the same time.” I asked him what he meant and he said, “Well, when I go to language arts, she’s teaching determining importance, and when I go to social studies, she’s teaching determining importance, and when I go to science, he’s teaching determining importance, only just in science, and when I go to math . . .” Ever prescient, I guessed it. “She’s teaching how mathematicians determine importance, right?” I asked. “Yep,” he said, with the smug satisfaction of having revealed a vast, school-wide conspiracy. I smiled and said, “Tony, my friend, it’s not a conspiracy, it’s called good planning. It’s called teaching in a way that kids will learn.” Disappointed, Tony turned away, shaking his head, and said, “Okay, I just thought you better know.”

If Comprehending Math were around when I was learning math, I might have become a lover of the science of patterns. If Comprehending Math had been around a few years ago, Tony would have had all his conspiracy theories realized much earlier in his education. If Comprehending Math had been around when I was teaching math, my students might have had a chance to do something other than watch me run around trying to do
a little bit of everything. They might have sensed a conspiracy. They might have made all-important thinking connections between and among the concepts that mattered most to them—throughout the day. They might have had more opportunities to use language, to create representations, to revise, to visualize as they approached math problems. They might have had a chance to understand their own thinking, not only about the books they read, but about the mathematical concepts I so wanted them to grasp. But Comprehending Math is here now and, for those who devour it, as I did, their teaching will be clearer, bolder, more connected. And for the ultimate beneficiaries, the help, they will have a chance to understand just how integrally our world is connected.

—Ellin Oliver Keene
This book has been a long time percolating. The coffee pot started up in 1970 when I was teaching six sections of general mathematics in an inner-city high school in Philadelphia. Can you imagine how deadening that course was to my students—the same arithmetic that had beaten them down for the previous seven years repeated again and again?

I was very fortunate to get to know two wonderful English teachers at that time who were experimenting with a “reading/writing workshop.” Yes, this was 1970. Lynne Miller is now a professor at the University of Southern Maine and Jolley Bruce Christman is the Founding Director of Research for Action, a research and evaluation organization in Philadelphia.

After a series of conversations and visits to each other’s classes we conducted a little experiment. In the next quarter, Lynne and I got back-to-back classes with the same students and scheduled them in for a double period with us to team-teach a reading, writing, and math workshop. It was a profound learning experience for me; it changed how I taught mathematics in several significant ways. I got my first taste of how reading, language, and writing should be taught. As a student, I was perpetually on the brink of disaster in my English classes from seventh grade on through college. I am sure that all my English teachers didn’t know what to do with me. I know I never quite understood what they wanted me to do. But when Lynne taught, she played with language and ideas in much the same way I played with mathematical materials and concepts. The powerful connections between language and thought that my psychology professors had emphasized suddenly became very real. Before reading, Lynne asked her students a million questions, some of which seemed pretty personal. She got them to imagine what the story was going to be about. Sometimes as they were reading she would ask them to stop and talk about what they saw in their minds. She got them to write! First they talked a lot about what they knew, what they thought, how they felt. Then they wrote heart-breaking stories about their lives.
While many of our contemporaries were preaching the need for “relevance” in the curriculum for inner-city kids, Lynne and Jolley had their students—regular inner-city high school students—reading Greek mythology. I remember them helping the kids study the Orpheus myth. A fancy, high-society women’s league was screening the film *Black Orpheus*, in which the myth is set in Brazil during Carnival. Lynne called and asked if they could bring about thirty high schoolers. The society ladies agreed but were a little nervous when the students arrived and all were black. To their credit, the ladies welcomed the students, and they all watched the film, totally entranced. When the film ended, the ladies embarked on their usual discussion of the film, the genre, the meaning of the symbols, and so forth, all done while tea and pastries were served. The students jumped right in and discussed their interpretations of the film, especially in light of the Orpheus legend they’d been studying. The ladies were impressed and as the students were leaving, the grand dame of the women’s league thanked Lynne for bringing her wonderful gifted class. Lynne laughed and assured her that this was just her regular third-period English class.

And that’s one of the key points in this book. Good teaching with powerful language and thinking is for all children. It is not just for the gifted; it’s for all the “regular third-period” kids that every teacher has.

Since that time with Lynne and Jolley, I have had the pleasure of collaborating with a number of truly fabulous reading, language, and writing teachers and professors: Rebecca Barr, Donna Ogle, Camille Blachowicz, Marilyn Bizar, Smokey Daniels, and Steve Zemelman. Along the way I have devoured the books written by Keene and Zimmermann, Harvey and Goudvis, and Miller, all from Denver’s Public Education and Business Coalition (PEBC). All of these people have helped me take the most powerful strategies for teaching reading comprehension, the development of language, and the process of writing and combine them with the cognitive approaches to mathematical problem solving that I love. That is what this book is all about.

Some early pieces of this approach to teaching mathematics can be seen in the book on problem solving that my wife and I wrote in 1991, *Mathwise*. As an elementary school teacher, Pam had encouraged me to see what teaching younger kids is all about. When I came to National Louis, I did just that. I taught my university courses in the evenings and spent many of my days in elementary school classrooms. What an eye-opening, mind-boggling experience! I could not possibly name all the wonderful teachers who have invited me into their classrooms to try my home-made manipulatives, activities, problem solving, and the different versions of the language/thinking/mathematics that I will share in this book. These teachers have helped me test and refine this approach over the past fifteen years. They incorporated my rough-hewn ideas and adapted them to work in their circumstances. They have given me feedback that I could share with others. Because they span the K–8 grades, I
can say that what follows works for students in that grade range. The examples in this book focus on grades one through six. It works for most of the kids most of the time when the teacher adapts it into her conscious repertoire of teaching mathematics. Nothing works all the time with every student. If any professor says his way of teaching math does, hold on to your checkbook.

My special thanks go to my wife Pam and all her third graders with whom I played math before she became a principal. She continues to be my touchstone for inspiration. And with great affection I must also thank Beverly Kiss, Mary Fencl, Carole Malone, Susan Gittings, Christina Hull, Michelle Habel, Katie George, Lynn Pittner, Cheryl Heck, and Shari and Pat Watson for sharing their teaching wisdom with me. Thanks to another fabulous teacher, Marie Bartolotta, for her insightful feedback on early chapters. I am very grateful for Emily Birch’s editorial skills and to both Leigh Peake and Emily for believing in this project and me. Thank you all.
INTRODUCTION

Braiding Mathematics, Language, and Thinking

[The] human ability—to imagine the future taking several paths, and to make adaptable plans in response to our imaginings—is, in essence, the source of mathematics and language. . . . [T]hinking mathematically is just a specialized form of using our language facility.


THREE KEY PIECES

I have said to many elementary school teachers over the past fifteen years, “If you have learned ways to help kids to think effectively and understand ideas in reading and language, use them in math and you won’t be disappointed.” These teachers know quite a bit about reading and language. This suggestion is a good one, but it is only part of the story. Certainly, if you use what you know works in reading, language arts, and writing to teach mathematics, you will get some good results. Teachers enthusiastically report significant involvement and understanding from their students. However, teachers can go beyond simply applying aspects of reading comprehension to mathematics. I have taught hundreds of teachers (and even reading specialists) in scores of courses and workshops how to do more through braiding together mathematics, language, and thinking.

If you set out to integrate mathematics and language (or problem solving and reading), what would you put together? What aspects of language fit nicely with mathematics? How about vocabulary? Or writing in math journals? However, the real question is what guides you in determining exactly what the kids should do and how you teach them. Asking the language arts people will only get you what they do in their knowledge domain. Concepts in mathematics are very different from concepts in language. They don’t really know the domain of mathematical knowledge or the process of mathematical thinking.

There is one major aspect of human thinking central to both language and mathematics: cognition. There are several key principles from cognitive psychology that can guide us. Therefore, I have used the term braiding to indicate that thinking, language, and mathematics can be braided together into a tightly knit entity like a rope that is stronger than
COMPREHENDING MATH

the individual strands. When these three important processes are braided, the result is stronger, more durable, and more powerful than any one could be by itself.

For many years, the teaching of mathematics and especially problem solving has suffered from insufficient attention to thinking and language. If you want students to understand mathematical ideas, they must use both language and thought. Trying to put more thinking into the math curriculum or one's teaching without attention to language will be fruitless and so will trying to use language without thinking. The term braiding here suggests that the three components are inseparable, mutually supportive, and necessary.

DEATH, TAXES, AND MATHEMATICS

There are two things in life that we can be certain of...everyone knows the answer...death and taxes. Add another item to that list: at least half of our nation’s fifth graders hate story problems. Actually, they dislike math in general, but story or word problems hold a special place of loathing in their souls. This percentage may vary a bit from classroom to classroom. Research has shown that most children start kindergarten with some fairly good ways to solve mathematical problems in the sandbox or with toys and games. However, during their first four or five years of school, they abandon their previously successful ways of dealing with problems involving mathematics.

Have you ever watched three children trying to figure out how to split up a bunch of candies? If the candies are identical, they just give one to each, then another, then another until they run out. Obviously influenced by Long John Silver of Treasure Island fame, they refer to this process as “divvying” the candies up. The children have not memorized division facts; they just have a way to do what is necessary. If the candies do not come out evenly (they often check to make sure everyone got the same number of candies), they may use some probabilistic device (e.g., flip a coin, odds or evens of total fingers displayed, paper/scissors/rock) to see who gets the extras. Children can be remarkably resourceful when no adults are around to tell them what to do.

No. I am not advocating Lord of the Flies. The question is: what forces are at work to dampen our children’s inherent awe and wonder, their excitement about learning, and their facility with mathematics? The early years of schooling present children with a torrent of messages that they must do math in one particular way, that there is one right answer, and one right way to find it. They are told what to memorize, shown the proper way to write down problems and answers in symbolic notation, and given a satchel full of gimmicks they don’t understand. No matter that they don’t understand what they are trying to memorize. Does anyone ask if the symbols make any sense to them? Does anyone notice that
they have ceased to trust their own reasoning or intuition borne of experience? Many children (and adults too) see mathematics as arcane, mystical. They believe that understanding mathematics is beyond most mere mortals. Can the children get the answer quickly, efficiently, and accurately? That's all that matters.

Of course, I believe that it is necessary for children to learn the basic math facts of the four operations. As much as I love problem solving, I know that students will be hampered in their problem solving (especially in estimation and determining reasonableness) if they don't know basic arithmetic facts. The question is not if those facts are learned, but how and when. All students should understand and be able to use number concepts, operations, and computational procedures. There are several critically important processes, each with a critical cognitive component, that lead to understanding, proficiency, and fluency that need to be developed. When students have many successful experiences using these processes, remembering math facts becomes a simple matter.

These processes are: counting (building one-to-one correspondence and number sense); number relations (decomposing and recomposing quantities to see relationships among the numbers); place value (creating sets of ten with objects and beginning to understand the base ten, positional notation); the meaning of the operations (creating mental maps of different situations and realizing that operations have multiple meanings); and fact strategies (thinking strategies for learning the facts for the operations).

Acting along with some erroneous beliefs about computation is another perhaps even more sinister force. Most people deny the importance of language in the world of mathematics. An exaggeration? Textbook publishers are very sensitive to the feedback from teachers whose message has been crystal clear for years: too many words on the page will make learning too hard for the kids who can't read well. “Johnny is not a good reader. Math is the only subject he likes (or does well in). Just let him work with the numbers.” So what happens when Johnny has to read the story problem? The teacher is ready with a magic trick: just look for the key word (cue word) that will tell you what operation to use. If you see sum or all together, you add the numbers. If you see take away or difference, then subtract the smaller number from the bigger number.

What is the fundamental message the kids get when told to look for the key/cue word? Don't read the problem. Don't imagine the situation. Ignore that context. Abandon your prior knowledge. Who cares about metacognition, metaphors, metamorphosis, metatarsals, whatever? You don't have to read; you don't have to think. Just grab the numbers and compute. After all, you've got a 25 percent chance of randomly selecting the correct operation.

This situation, all too prevalent in U.S. schools, discourages kids from thinking. That makes no sense. Both reading and mathematics require thinking. Teachers should use every means possible to encourage
students to think, reflect, question, imagine. And how do we do that? With language—with expressive language (speaking and writing) and with receptive language (listening and reading). And they all fit together in a child’s life and in the classroom.

There is yet another critical reason to braid language, thinking, and mathematics. When math is taught with the language pruned or purged, who is immediately penalized? Those who use language as their primary means of processing ideas; those who develop their language facility early. Girls. Of course, all children can profit from discussing, verbalizing thoughts, talking mathematics, but girls develop language strengths earlier than boys and, when encouraged, can use them effectively to build mathematical understanding. Girls understand the value of braiding language, thinking, and mathematics; they even get the metaphor.

A LITTLE FORESHADOWING

Consider the following ideas, each culled from the literature on reading and language learning and well known to most elementary school teachers. Each of these ideas has a solid foundation in cognition. I have simply played those ideas through mathematics and problem solving. Like the television game show, Jeopardy, I have worded the idea in the form of a question.

1. Are students expected to construct their own meaning in mathematics?
2. Are students encouraged to have ownership of their problem solving—to choose to use mathematics for purposes they set for themselves? What would ownership look like?
3. Are students encouraged to do problem solving for authentic purposes? What would authentic mathematics look like?
4. Are students encouraged to do voluntary mathematics, selecting tasks for information, pleasure, or to fulfill personal goals?
5. How is mathematics instruction scaffolded?
6. Does the school help teachers and students build a rich, mathematically literate environment or community?
7. Are students encouraged to see the big picture, important concepts, vital connections versus isolated pieces of mathematics?
8. Is forgiveness granted to students in mathematics? Is making mistakes a natural part of learning? Is doing mathematics seen as a dynamic process that incorporates planning, drafting, revising, editing, and publishing?

Is it heresy for an unapologetically passionate teacher of mathematics to believe that we could do a far, far better job of teaching children how to understand and love mathematics if we did all of these things?
What is reading? Sounds like a silly question, but if you have been following the “reading wars” in the past twenty years (and you may have also followed the “math wars”), there are a whole lot of people who believe that reading is decoding, phonics, and word attack skills. No respectable educator would argue that these things are not part of the process of reading and learning to read, but they do not define it. By analogy, arithmetic computational proficiency and math facts are part of mathematics, but they do not define it. Mathematics is the science of patterns. Neither reading nor math is a collection of skills or subskills. I have no intention of addressing all aspects of reading and language in this book, nor should I. Instead, I will draw selectively from the experts whom I admire.

Reading is the process of constructing meaning from written language. Reading is thinking. Constructing meaning does involve decoding, but in greater measure, it is a very dynamic process requiring some very special thinking about what one knows already (prior knowledge) and one’s experiences (especially with language). Readers interact with what they read. They do not passively receive its meaning, they construct it. They use what they know about the content of the text, about the context being described, about how texts of this kind are structured (their format), and about the particular vocabulary (including specialized terms). They must continually draw inferences about the meaning of the words. They must make assumptions about missing pieces, things implied but not there on the page. For proficient readers, this is all done effortlessly and largely unconsciously. As complex as all these processes are, throw into the mix that these things are greatly facilitated by the readers’ metacognitive monitoring of what they are doing and a metacognitive awareness of their own ways of operating, being able to reflect on their own ways of thinking (as if looking at your mind in a four-dimensional mirror).

Some people ask: “Do children really do all those things? Don’t they just learn phonics and listen to people talk and they figure out how to read?” In psychology, they call that kind of thinking “the black box.” We don’t know what goes on inside kids’ heads. It is a black box. We can’t read their minds. We can only go by their behavior. Such an approach is dangerous. And yet, I hear it frequently in both reading and math. A significant portion of adults in this country believe that if they could just memorize the math facts, they’d be fine in subsequent mathematics courses. Ironically, some people who are involved in one of the biggest “reform” curricula in math have said that if teachers just give the kids some good math to do, by “osmosis” they will construct meaning. OSMOSIS!

Those who believe in such osmosis may lack an understanding of cognition and metacognition, and how a teacher can facilitate meaning-making. Teachers model, show, ask questions, make suggestions, and
create a safe, supportive, rich, literate environment in which students can explore ideas and interests. Teachers also can mediate between the larger world and the world of the child. Sometimes they explain things. Language, especially oral language, is used continually throughout these processes.

**Reading Comprehension Strategies**

Research on reading has identified several highly effective cognitive strategies for students to use in reading comprehension. Specific teaching techniques for helping students with these strategies have been developed. With minor differences in terminology among experts in the field, they are:

- **making connections** (activating relevant prior knowledge, linking what is in the text to their own experiences, discerning the context; relating what is in the text to other things they’ve read, things in the real world, to phenomena around them);
- **asking questions** (actively wondering, raising uncertainties, considering possibilities, searching for relationships, making up “what if” scenarios);
- **visualizing** (imagining the situation or people being described, making mental pictures or images);
- **inferring and predicting** (interpreting, drawing conclusions, hypothesizing);
- **determining importance** (analyzing essential elements);
- **synthesizing** (finding patterns, summarizing, retelling);
- **metacognitive monitoring** (actively keeping track of their thinking, adjusting strategies to fit what they are reading).

When teachers focus on these cognitive strategies in a variety of different text genres, students can learn to use those strategies independently and flexibly. The cognitive strategies are taught most effectively in a reading workshop that includes (1) crafting lessons with direct, explicit instruction and modeling by the teacher, (2) students applying the content of the crafting lesson, and (3) students reflecting at the end of the reading workshop (Public Education and Business Coalition [PEBC] 2004).

During crafting lessons, teachers introduce and explain a new strategy. They think aloud as they read, modeling their own use of that strategy for their students and carefully explaining how they are applying the new strategy to the text. After the crafting lesson, students spend large amounts of time applying the content of the crafting lesson to their own reading experiences. During this time, students might meet in small, needs- or interest-based groups, or read independently. Teachers spend this time guiding small groups of students as they negotiate a common text or a common instructional need, or conferring with individuals as they work to make sense of their reading materials. At the end of the reading workshop, students regularly share their insights about the content of their reading and their use of reading strategies. The format for
this reflection can vary, depending on purpose. The teacher participates in the reflecting, offering observations and recording the individual and group needs generated by this process. The goal is for students to internalize these strategies and use them easily.

These seven strategies are fairly broad and incorporate quite a few pieces. Reading experts have also developed more focused strategies. For instance, K-W-L (Know-Want to know-Learn) is a method of having kids think about key ideas before, during, and after reading. QAR (Question-Answer-Relationship) is a method of asking questions while reading. Each chapter of the book addresses one of the broader reading comprehension strategies listed above. It is not my intention to be comprehensive. There are many marvelous books cited in the references list at the end of the book that provide a wealth of examples. In each chapter, I have tried to include only the key elements of reading and language, identified by principles of cognition, that can be braided with mathematics. The chapters will be both cumulative and recursive. Later chapters will incorporate previous ideas into the main strategy being discussed and also will revisit previous ideas to build a deeper understanding of them, in light of the new strategy.

**MATHEMATICAL THINKING AND PROBLEM SOLVING**

The National Council of Teachers of Mathematics (NCTM) developed a set of principles and standards for mathematics curriculum, teaching, and assessment in 1989. NCTM produced a revised version in 2000 with standards addressing five broad strands of mathematics K–12: Number and Computation; Algebra; Geometry; Measurement; Data and Probability. Each of the strands is chock full of powerful mathematical concepts. In order for students to learn those concepts with deep understanding, the NCTM Standards address five processes in which students must be engaged: problem solving, connections, reasoning and proof, communication, and representations.

Despite their complexity, I infer that the NCTM standards are based on three big (that is, foundational) ideas:

1. Math is the science of patterns; it is much more than arithmetic. Every strand of math has certain patterns that we look for. Probably every concept in mathematics is a pattern of some kind.
2. The goal of mathematics teaching should be understanding concepts, not merely memorizing facts and procedures. Therefore, we must use what we know about cognition.
3. For children to understand mathematical concepts, they must use language, the quintessential characteristic of human cognition.

The five content standards have provoked a lot of dialogue about the concepts of the elementary school curriculum. Less progress has been made regarding the process standards. NCTM views problem solving as
doing mathematics and as a powerful vehicle for building understanding of mathematical concepts. This would be a shocker for most of my old math teachers, for whom problem solving was an afterthought, something that the kids did after they’d been taught the concept or procedure. Today we can see that by using well-constructed problems, worthwhile mathematical tasks, the use of good strategies, and with the teacher’s facilitation, students can construct deeper meaning for concepts by actually using the mathematics they know.

 Granted, the majority of classrooms in the United States may be using textbooks that are hanging on to a conception of problem solving as determining what computational procedure to use. The problems may be reminiscent of the settlers leaving Missouri bound for Oregon. For example, “Hattie needs 5 and 7/8 yards of muslin at 27 cents per yard. What will it cost?” They are generally called “routine” problems or “translation” problems (translating the description of the situation into an equation). Somewhat more complex story problems usually entail a string of consecutive computational procedures in order to find the correct answer.

In the 1950s George Polya helped to broaden our sense of problem solving by describing heuristics or strategies that college students could use in their mathematics classes. By the early 1980s the idea of strategies found its way into the school curriculum and most textbooks introduced students to a dozen or so problem-solving strategies. A problem was seen as a task for which the person confronting it wants to find a solution, but for which there is not a readily accessible procedure that guarantees or completely determines the solution. Consider the case of a student confronted with the question, “Sacks of flour cost $4.85 per sack; how much would you pay for ten sacks?” If this student understands that there are ten sacks and each one costs the same amount ($4.85) and realizes that the answer can be readily determined by repeated addition or by multiplication, is this a “problem” for her or him? No, it is a thinly disguised drill exercise, perhaps valuable, but not a problem.

A number of math educators have infused textbooks with problems that require students to do more than merely determine which operation(s) to use, moving beyond what educators see as translation problems. They created “nonroutine” problems, what some called “process” problems, in which a good process of thinking was required. For instance, “How many different ways can a person make change for a quarter?” There is no obvious computation procedure to invoke. You have to figure it out. Try it.

If you tried it, did you find twelve different ways? But how did you work it? Did you make a list? Did you draw a picture? Did you make a table? If so, you chose a problem-solving strategy. Strategies can help students find solutions. They may also help them understand the problem.

In the 1980s some educators came to think of problem solving as an “art” in which mathematicians (as well as regular humans) worked on perplexing problems. They placed problem solving at the heart of mathematics. For many of these educators problem solving required working
from an “initial state” to a “goal state” and strategies were to be used when one was “stuck” and did not know what to do next to move toward the end goal.

A serious drawback to this view is that it treats problem solving as a process independent of content. Strategies tend to be seen as generic, applicable to anything, and able to be mastered, like a skill or a procedure. For me, the term problem-solving skill is an oxymoron. Skills are physical in nature, requiring a certain amount of innate ability and massive amounts of practice, but with minimal thought or reasoning. In contrast, problem solving is clearly a cognitive venture. How you think and what you think about are intimately related. Analyzing a poem and analyzing a spreadsheet of data are very different processes. That they are similar in that both “break things down” becomes completely irrelevant when one is immersed in the task.

There is yet another way that some math educators are conceiving of problem solving and strategies. Some use a perspective referred to as modeling or creating models in which problem solving serves primarily to interpret the problem. Similar to what some would call task definition, this broader approach to problem solving emphasizes the need for interpretation, description, elaboration, and explanation of the nature of the problem. This perspective recognizes the importance of the context, content, and the concepts of the problem. The solution to problems is often the building of a model using particular concepts that are still being developed by the students. In this view, the purpose of the strategies is to help students refine, revise, and extend their ideas, especially through interaction with others.

The point is the kids have to do the math. How does a student get better at solving problems? What is the best way to get better at reading? By reading more. Of course, a third grader can’t simply pick up Kant and make any sense out of him. (Come to think of it, I can’t either.) Problems should be challenging, but not overwhelming.

The dilemma goes even deeper. The NCTM process standard termed Connections encourages a wide variety of links within mathematics. For decades the mathematics curriculum has consisted of little, bite-sized chunks of mathematical knowledge. I am not speaking only of narrowly defined skills (as in skill and drill), although some still cling to the erroneous belief that if children crank out a gazillion math facts they know how to do mathematics. I am concerned here with the fragmentation of concepts into isolated compartments that is contrary to the NCTM admonition that students need to see that mathematics is a coherent whole. Many concepts are connected to a multitude of others. Even when teachers go after conceptual understanding, the curriculum treats that concept in isolation from its related concepts. To make matters worse, the curriculum deals with a topic for two weeks and then ignores it for a year.

My wife and I always knew when it was April 1 because the fraction worksheets would come home to be stuck on the refrigerator with magnets. That would go on for two weeks and then as suddenly as they had
come, they just disappeared one day (usually at the deadline for filing income tax returns—I am not sure what the connection is). What are kids to make of this phenomenon? Fractions only exist during these two weeks? Nobody thinks about them or uses them at any other time during the year.

What about the highly touted “spiraling” curriculum, which does not expect mastery (or understanding?) the first time a child encounters a concept because it will be back two more times during the year? The issue is not how many times or how often one revisits a concept, but the nature and quality of the experience. Is it conceptually rich? Have the students built a solid initial foundation to begin using the concept in a way that is meaningful to them? Some spiraling math curricula are more like tornados. What happens when the tornado touches down? It briefly stirs things up, and then leaves for an indeterminate time. If its touchdown time is riddled with gimmicks, what do the kids have to show for their brief encounter with the mathematics of that moment?

Another NCTM process standard not fully developed in the United States is representations. It was barely mentioned in the 1989 version of the standards. Therefore, in 1991 my wife and I wrote Mathwise, in which we made a strong pitch for the critical importance of creating representations in doing mathematics. Furthermore, we asserted that of the ten popular problem-solving strategies, there are five strategies that are based on representations, two that were so broad as to be metastrategies that should be used all the time, and three that were fairly narrow and should be considered supplementary.

When using the five most powerful strategies, students create their own representations. Through this creation, they are truly constructing meaning. These five strategies are:

- Discuss the problem in small groups (language representations using auditory sense).
- Use manipulatives (concrete, physical representations using tactile sense).
- Act It Out (representations of sequential actions using bodily kinesthetic sense).
- Draw a picture, diagram, or graph (pictorial representations using visual sense).
- Make a list or table (symbolic representations often requiring abstract reasoning).

Language should be used throughout all five of these strategies.

The two common strategies of looking for a pattern and using logical reasoning always should be used in problem solving. Mathematics is the science of patterns; every branch of mathematics (e.g., numbers, geometry, measurement, data and chance, algebra) has characteristic patterns. Logical reasoning is essential to doing mathematics. But is it a strategy? Is it something one chooses to do instead of something else? Okay. What is the alternative? “Pay attention, kids; we’ve been reasoning illogically all
year long in math; now it is time for a new strategy. Let's use logical reasoning for a change!"

Children delight in seeing patterns in mathematics or truly understanding a concept. Consequently, I am concerned when children's books present seeing mathematical patterns everywhere as a "curse," mathematics as magic or as witchcraft practiced by the Number Devil. I believe it is a big mistake to tell children that mathematics is magical or incomprehensible while at the same time trying to help them believe in their own capabilities and that they can expect it to make sense through diligent work. The books may be cute, but can send a decidedly mixed message.

When students are taught how to look for patterns and reason logically in every activity along with the five representational strategies, the representations they create build understanding of the problem (and lead to a solution). In creating them, students are developing different mental models of the problem or phenomena. In rich, meaningful mathematical tasks, students may use several of these representations, moving from one to another to figure out more about the problem. Later they might draw on supplementary strategies (such as the three popular ones: guess and check, work backwards, simplify problem), but these cannot be used effectively unless one understands the problem. As students become more mathematically sophisticated, they are able to use more abstract and symbolic strategies (e.g., use proportional reasoning, apply a formula).

Obviously, thinking is critically important in reading and language as well as mathematics and problem solving. It is beyond the scope of this book, let alone this chapter, to adequately address cognition or all the related cognitive issues. Fortunately two wonderful volumes do a fine job of just that. They are How People Learn (Bransford, Brown, and Cocking 2000) and its companion, How Students Learn (Donovan and Bransford 2005), which uses three main principles to synthesize a tremendous amount of information about human cognition:

1. engaging prior understandings (using prior knowledge, confronting preconceptions and misconceptions)
2. organizing knowledge (developing a deep foundation of factual knowledge organized into coherent conceptual frameworks that reflect contexts for application and knowing when to use which information—referred to as conditionalized knowledge)
3. monitoring and reflecting on one's learning (developing metacognitive processes and self-regulatory capabilities)

HOW DO ALL THESE IDEAS FIT TOGETHER?

Fitting all these ideas together is not an easy task. May I phone a friend? In Figure I.1, I have simply placed the major ideas that need to be braided into three separate clouds. Imagine that each cloud was an overhead transparency that could be placed like a template on top of one of the other clouds. What connects to what?