Introduction to Reasoning and Proof

Grades PreK–2

Karren Schultz-Ferrell
Brenda Hammond
Josepha Robles

The Math Process Standards Series
Susan O'Connell, Series Editor

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CONTENTS

FOREWORD

In order to be effective mathematicians, students need to develop understanding of critical math content. They need to understand number and operations, algebra, measurement, geometry, and data analysis and probability. Through continued study of these content domains, students gain a comprehensive understanding of mathematics as a subject with varied and interconnected concepts. As math teachers, we attempt to provide students with exposure to, exploration in, and reflection about the many skills and concepts that make up the study of mathematics.

Even with a deep understanding of math content, however, students may lack important skills that can assist them in their development as effective mathematicians. Along with content knowledge, students need an understanding of the processes used by mathematicians. They must learn to problem solve, communicate their ideas, reason through math situations, prove their conjectures, make connections between and among math concepts, and represent their mathematical thinking. Development of content alone does not provide students with the means to explore, express, or apply that content. As we strive to develop effective mathematicians, we are challenged to develop both students' content understanding and process skills.

The National Council of Teachers of Mathematics (2000) has outlined critical content and process standards in its *Principles and Standards for School Mathematics* document. These standards have become the roadmap for the development of textbooks, curriculum materials, and student assessments. These standards have provided a framework for thinking about what needs to be taught in math classrooms and how various skills and concepts can be blended together to create a seamless math curriculum. The first five standards outline content standards and expectations related to number and operations, algebra, geometry, measurement, and data analysis and probability. The second five standards outline the process goals of problem solving, reasoning and proof, communication, connections, and representations. A strong understanding of these standards empowers teachers to identify and select activities within their curricula to produce powerful learning. The standards provide a vision for what teachers hope their students will achieve.

This book is a part of a vital series designed to assist teachers in understanding the NCTM Process Standards and the ways in which they impact and guide student learning. An additional goal of this series is to provide practical ideas to support teachers as they ensure that the acquisition of process skills has a critical place in their math instruction. Through this series, teachers will gain an understanding of each process standard as well as gather ideas for bringing that standard to life within their math classrooms. It offers practical ideas for lesson development, implementation, and assessment that work with any curriculum. Each book in the series focuses on a critical process skill in a highlighted grade band and all books are designed to encourage reflection about teaching and learning. The series also highlights the interconnected nature of the process and content standards by showing correlations between them and showcasing activities that address multiple standards.

Students who develop an understanding of content skills and cultivate the process skills that allow them to apply that content understanding become effective mathematicians. Our goal as teachers is to support and guide students as they develop both their content knowledge and their process skills, so they are able to continue to expand and refine their understanding of mathematics. This series is a guide for math educators who aspire to teach students more than math content. It is a guide to assist teachers in understanding and teaching the critical processes through which students learn and make sense of mathematics.

Susan O'Connell
Series Editor

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Finally, we wish to applaud our complete confidence in one another. Our ability to approach and complete this project with humor, honesty, and mutual respect saw us through this exciting project and kept our friendship thriving.

The Reasoning and Proof Standard

The ability to reason systematically and carefully develops when students are encouraged to make conjectures, are given time to search for evidence to prove or disprove them, and are expected to explain and justify their ideas.

—National Council of Teachers of Mathematics,
Principles and Standards for School Mathematics

Why Focus on Reasoning and Proof?

The mathematics classroom of the past might have been one of the quietest places in the school. Opportunities for students to explain thinking and reasoning were rarely offered. In today's classrooms, however, reasoning is viewed as a necessary process for ensuring that students understand mathematics concepts and skills. "Many mathematicians consider the NCTM standard concerning reasoning and proof to be its most important" (Chapin, O'Connor, and Canavan Anderson 2003, 78). Students benefit when we provide them with opportunities to explain thinking and reasoning not only through discourse but also through the recording of representations (e.g., charts, graphs, drawings, diagrams, etc.) of mathematical thinking. Students should also be given opportunities to reflect on their thinking and reasoning through writing. All of these experiences help students extend their thinking, solidify understandings about concepts and skills, and learn the different ways their classmates think about, reason with, and solve mathematics problems and situations.

Although we realize the importance of questioning our students about their thinking when problem-solving errors are evident, many of us do not realize how critical it

is to question students about their mathematical thinking and to discuss their reasoning with them at all times. Listening to students as they explain their reasoning in thinking about a mathematical idea gives us valuable information about what they know and are able to do. Students' strengths and misconceptions are revealed, which provides us with information that will help us plan instruction to meet the needs of all of our students. In addition to helping us gain information about students, our questioning delivers important messages to students: their ideas are valued and important; mathematics is less about memorization and more about reasoning; and what they are learning should make sense (Burns 1997).

As previously noted, reasoning can be expressed through representations as well. We must encourage young children to record how they are thinking about the math they are learning by drawing representations of these ideas. These representations become evidence of learning that can be shared later with families. When students become confident in explaining and representing thinking and reasoning, then they are more able to write about their reasoning. When we emphasize reasoning in our classrooms, then our students will become engaged in different types of reasoning, and they will begin to understand how to express acceptable mathematical explanations.

What Is the Reasoning and Proof Standard?

The National Council of Teachers of Mathematics (NCTM) has recommended standards that can be used as a resource and a guide for teachers as we plan and create instructional lessons and activities to develop our students' understandings about mathematics. The first five standards are the mathematical content goals: number and operations, algebra, geometry, measurement, and data analysis and probability. The next five standards address the processes by which students explore and use mathematics as well as develop understandings about mathematics concepts and skills. The process standards should be embedded throughout our instructional program.

> Instructional programs from prekindergarten through grade 12 should enable all students to—
>
> - recognize reasoning and proof as fundamental aspects of mathematics;
> - make and investigate mathematical conjectures;
> - develop and evaluate mathematical arguments and proof; and
> - select and use various types of reasoning and methods of proof. (NCTM 2000, 56)

This book explores ways to assist young learners in extending their thinking, deepening understandings, and making sense of mathematics through the process standard of reasoning and proof.

Developing Skills and Attitudes

"Reasoning mathematically is a habit of mind, and like all habits, it must be developed through consistent use in many contexts" (NCTM 2000, 56). The ability to reason develops over time with multiple experiences. We must provide our students with daily opportunities to reason about the mathematics they are learning. It must become a part of the way we teach in order for our students to become proficient in their ability to reason.

Establishing a classroom environment that is both respectful and supportive is the first step in helping our students become comfortable enough to offer explanations of mathematical thinking and reasoning with confidence. Ground rules are needed to create a classroom climate in which students listen respectfully to one another during mathematics discussions. Students must feel safe when expressing their thinking and sharing their own ideas for examination. They will not contribute to a discussion if they feel their ideas will be made fun of or dismissed quickly. We must be consistent with our expectations of behaviors during these discussions.

This also involves the expectation that all students will listen to what others say, which allows them to participate more actively in an ongoing discussion. Students need our support in order to develop the skills necessary to participate in teacher-to-student, student-to-student, and student-to-teacher discourse. They must have daily opportunities to talk and reason about mathematics if we want them to be able to then represent and write about mathematical reasoning.

Presenting tasks to students that are open-ended and rich in context provides students with opportunities to develop reasoning skills. When we present problems to our students that can be solved in more than one way, or that have more than one correct answer, it promotes talk about thinking and reasoning. Problems that cause students to stretch their thinking are a must. This book addresses all the considerations introduced in this section, as well as others, to help you begin to implement the reasoning and proof standard into your daily instruction.

How This Book Will Help You

This book is designed to help you better understand the NCTM Process Standard of reasoning and proof for students in prekindergarten through grade 2. It explains how these students' early reasoning abilities prepare them for more complex reasoning in the intermediate grades and beyond. Specific types of reasoning are described, and student dialogues are provided throughout the book to model how primary students use these different ways to reason. The teachers facilitating these student dialogues model critical questioning that promotes rich discussions in which students are reasoning about the mathematics they are learning.

We share tools and strategies that will help your students think deductively and inductively in an organized manner. These tools and strategies will support students in solving complicated problems and are included for you on the CD. We discuss how you can provide reasoning experiences for your students in which they will begin to make

conjectures about important mathematical ideas and relationships. Primary students should be encouraged to support their conjectures by providing explanations and justifications. Student dialogues are provided to demonstrate how children begin to formulate conjectures and then explain their reasoning with informal mathematical arguments.

We note several strategies that teachers can use to support students in developing reasoning skills, and activities that promote reasoning are included on the CD. The activities provided on the CD do not list specific grade levels because the activities have been formatted so that you can modify them to meet the needs, interests, and skills levels of all your students. We describe how to do this in the "About the CD-ROM" section near the end of this book. We encourage you to choose activities that fit your content and instructional goals. Many of the problems and activities demonstrated for you in the student dialogues throughout the book are on the CD. You will find these activities to be both engaging and challenging for your students, and facilitating discussions with your students about the problem solving within the activities will support their reasoning skills. These discussions can occur either while your students are solving the tasks as a whole group or by discussing students' problem solving and reasoning after they have completed an activity. In addition to the CD activities, you will find a listing of resources that will help your students make sense of mathematics, promote reasoning, and strengthen their mathematics concepts and skills. Also included on the CD are several rubrics that will be helpful in assessing your students' development of reasoning skills: a rubric you can use to monitor your students' development of reasoning skills and a rubric students can use to monitor their reasoning progress.

As mentioned earlier, examples of student discourse demonstrate how young students reason about important mathematical ideas and relationships and serve as a model for how to facilitate discussions that promote reasoning in your own classroom. These examples demonstrate how students are reasoning about a variety of mathematics concepts and skills. In addition, examples of student work are presented that provide another glimpse into students' thinking as their reasoning and proof skills develop. Practical tips and ideas are shared in the "Classroom-Tested Tip" boxes to help you implement the ideas explored in the chapters. Following each chapter, you will find several discussion questions included for you to reflect on the content of the chapter, either alone or with a group of your colleagues.

Chapter 5 discusses how the process standard of reasoning and proof is supported by the remaining process standards: problem solving, communication, representation, and connections. We discuss how students' reasoning helps them make vital connections to their everyday lives, other concepts in math, as well as other discipline areas such as reading, science, and social studies. We share ideas for how to provide students with meaningful tasks that connect reasoning to all the process standards. In Chapter 6, a variety of ways to assess students' reasoning skills are examined. After we have explored the reasoning and proof standard in depth, you will see how it relates to the different content areas in mathematics in Chapter 7, "Reasoning and Proof Across the Content Standards."

The role of communication in supporting students' reasoning has been stressed several times in this chapter, especially communicating reasoning through discourse. It is important to remember that we cannot expect our students to write about math-

ematical reasoning if they have not talked about it first. Students' ability to communicate their thinking and reasoning through writing as well as through pictorial and symbolic representations is examined in various chapters, and examples of student work have been included throughout the book.

We hope this book will enhance your understanding of the reasoning and proof standard and provide you with practical ideas and strategies that will help develop your students' reasoning and proof skills. When we grow in our understandings about reasoning and proof, so will our students.

> Students who leave the elementary grades with a mathematics education that has focused on mathematical reasoning are students who can count on their own thinking and are willing and able to investigate new problem situations for themselves. (Stiff 1999, 12)

Questions for Discussion

1. Did mathematics make sense to you when you were in school? Were you ever asked to explain how you solved a problem? How could your past experiences and attitudes about reasoning affect how you think young students are able to reason?

2. If students state the correct answer but cannot explain why the answer makes sense mathematically, what questions might you have about your students' understanding? How might this inability to explain and reason about mathematics present difficulties later in school?

3. What attitudes or dispositions are crucial for explaining one's thinking and reasoning? How does the atmosphere in the classroom affect students' abilities to develop reasoning?

4. Reflect on the quote included at the end of this chapter by yourself or with colleagues. Compare students who learn mathematics through the process of reasoning to students who learn mathematics through only procedures. How are the two types of learning different? How might students' understanding of mathematics be affected if they only learn procedurally?

Lupita's Shapes

The Task:

In this experience students will:

- describe and compare attributes of shapes
- reason deductively

Materials:

- circles, quadrilaterals (trapezoids, rectangles, squares, parallelograms, and rhombi), different sizes and types of triangles from the toolkit
- pencils, one per student
- crayons

Directions:

1. Pose the following problem to students:
   Lupita colors a shape in class. Listen to the clues to figure out the shape Lupita colors.

   Clues:
   - Lupita's shape has straight lines.
   - Lupita's shape has three sides.
2. Give students shapes to figure out the problem as the clues are read.

Talk About It:

Try these questions to stimulate thinking and discussion:

- Tell one thing about the shapes you see.
- How do you know this shape is a triangle?

Write About It:

Ask students to draw a circle, a triangle, and a square. Then ask them to draw a loop around their favorite shape and write one thing about this shape.

Tiered Learning:

Adjust or extend the activity in the following ways:

- Make a circle in the air with your finger. Repeat with a triangle.
- Make a large circle on the floor with yarn. Students walk around the perimeter of a circle. Repeat with triangles and quadrilaterals.
- Locate shapes in the classroom and describe them.
- Show students two different quadrilaterals (e.g., trapezoid and rectangle). Ask students, "How are these shapes alike? How are they different?"

Name _____

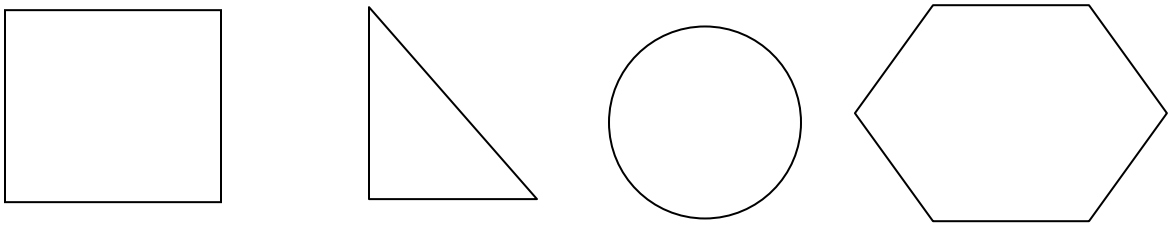
Tanya's Shape

Tanya draws shapes on her paper in art class.

Use the shapes and the clues to figure out which shape is Tanya's favorite.

Clues:

- My shape has fewer than 6 sides.
- My shape has only straight lines.
- My shape has more than 3 sides.



Draw Tanya's favorite shape below. Tell how you know your answer is correct.

_____

_____

_____

_____