Introduction to Problem Solving

Grades PreK–2

Susan O'Connell

The Math Process Standards Series
Susan O'Connell, Series Editor

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FOREWORD

In order to be effective mathematicians, students need to develop understanding of critical math content. They need to understand number and operations, algebra, measurement, geometry, and data analysis and probability. Through continued study of these content domains, students gain a comprehensive understanding of mathematics as a subject with varied and interconnected concepts. As math teachers, we attempt to provide students with exposure to, exploration in, and reflection about the many skills and concepts that make up the study of mathematics.

Even with a deep understanding of math content, however, students may lack important skills that can assist them in their development as effective mathematicians. Along with content knowledge, students need an understanding of the processes used by mathematicians. They must learn to problem solve, communicate their ideas, reason through math situations, prove their conjectures, make connections between and among math concepts, and represent their mathematical thinking. Development of content alone does not provide students with the means to explore, express, or apply that content. As we strive to develop effective mathematicians, we are challenged to develop both students' content understanding and process skills.

The National Council of Teachers of Mathematics (2000) has outlined critical content and process standards in its *Principles and Standards for School Mathematics* document. These standards have become the roadmap for the development of textbooks, curriculum materials, and student assessments. These standards have provided a framework for thinking about what needs to be taught in math classrooms and how various skills and concepts can be blended together to create a seamless math curriculum. The first five standards outline content standards and expectations related to number and operations, algebra, geometry, measurement, and data analysis and probability. The second five standards outline the process goals of problem solving, reasoning and proof, communication, connections, and representations. A strong understanding of these standards empowers teachers to identify and select activities within their curricula to produce powerful learning. The standards provide a vision for what teachers hope their students will achieve.

This book is a part of a vital series designed to assist teachers in understanding the NCTM Process Standards and the ways in which they impact and guide student learning. An additional goal of this series is to provide practical ideas to support teachers as they ensure that the acquisition of process skills has a critical place in their math instruction. Through this series, teachers will gain an understanding of each process standard as well as gather ideas for bringing that standard to life within their math classrooms. It offers practical ideas for lesson development, implementation, and assessment that work with any curriculum. Each book in the series focuses on a critical process skill in a highlighted grade band and all books are designed to encourage reflection about teaching and learning. The series also highlights the interconnected nature of the process and content standards by showing correlations between them and showcasing activities that address multiple standards.

Students who develop an understanding of content skills and cultivate the process skills that allow them to apply that content understanding become effective mathematicians. Our goal as teachers is to support and guide students as they develop both their content knowledge and their process skills, so they are able to continue to expand and refine their understanding of mathematics. This series is a guide for math educators who aspire to teach students more than math content. It is a guide to assist teachers in understanding and teaching the critical processes through which students learn and make sense of mathematics.

Susan O'Connell
Series Editor

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The Problem-Solving Standard

Solving problems is not only a goal of learning mathematics but also a major means of doing so.

—National Council of Teachers of Mathematics,
Principles and Standards for School Mathematics

Why Focus on Problem Solving?

Traditionally, problem solving was viewed as a distinct topic, introduced to students after they had mastered basic skills. In today's classrooms, however, problem solving is recognized as the central focus of mathematics instruction. The ability to solve problems is the ultimate goal of mathematics. It is why we teach students to add, subtract, multiply, and divide. It is why we teach them to work with fractions, measurement, and geometry. Our goal is not for students to perform isolated computations, but rather to be able to apply their varied math skills to solve problems. But problem solving is more than just a goal of learning mathematics, it is also a critical process, woven across the entire mathematics curriculum, through which students are able to explore and understand mathematics (NCTM 2000, 52). Through problem-solving experiences, students learn to challenge their thinking about data and probability, test their ideas about numbers and operations, apply their skills in geometry and measurement, and evaluate their understandings of algebra. Through problem-solving tasks, students develop an understanding of math content and ultimately use that content understanding to find solutions to problems. Problem solving is both the process by which students explore mathematics and the goal of learning mathematics.

One objective of problem-solving instruction is to enable students to use their repertoire of math skills to solve problems. But it takes more than isolated math skills

to be an effective problem solver. It also takes a variety of thinking skills that allow students to organize ideas, select appropriate strategies, and determine the reasonableness of solutions. It takes an understanding of how to use and adapt strategies to fit the problem situation. And it takes an ability to reflect on how we solve problems to help us better understand our own thought processes and identify why we select and apply various strategies.

In the past, problem solving may have been viewed as an isolated assignment (e.g., a list of word problems), but today problem solving has an integrated role in the math classroom. Teachers begin lessons by posing a problem, then skills and strategies are developed throughout the lesson as the problem is explored, and those newly acquired skills allow students to successfully find a solution. Problem solving becomes both the starting point and the ending point to well-balanced mathematics lessons. Developing students' computational skills is important, but teaching those skills in a problem-solving context ensures that students not only understand the skill but see the meaningfulness of learning the skill and understand how to apply it to real-world situations. "Problem solving is the process by which students experience the power and usefulness of mathematics in the world around them" (NCTM 1989, 75).

What Is the Problem-Solving Process Standard?

The National Council of Teachers of Mathematics (NCTM) has developed standards to support and guide teachers as they develop classroom lessons and create activities to build their students' mathematical understandings. Some of those standards delineate the content to be addressed in the math classroom, while other standards address the processes by which students explore and use mathematics. Problem solving is a critical math process, and the components of the NCTM Problem Solving Process Standard reflect its complex nature. Instructional programs (NCTM 2000, 52) should enable students to:

- build new mathematical knowledge through problem solving;
- solve problems that arise in mathematics and in other contexts;
- apply and adapt a variety of appropriate strategies to solve problems;
- monitor and reflect on the process of mathematical problem solving.

Throughout this book, we explore ways to assist primary students in building new math knowledge through problem-solving tasks. Highlighted problem-solving activities may be presented in math contexts as well as real-world contexts. We explore, in depth, the various problem-solving strategies that support young students in finding solutions, and we identify techniques for helping our students reflect and monitor their problem solving. We will dive into the NCTM process standard of problem solving in order to better understand it and find ways to bring it to life within our primary classrooms.

Creating Effective Problem Solvers

In my early experiences with teaching problem solving, I began much as my own teachers had: I assigned problems to students and expected them to be able to solve the problems on their own. I quickly recognized my students' anxiety and frustration. I soon learned that assigning problems and then correcting those problems did not create successful problem solvers. I began to break down the skills needed to solve problems and find opportunities to guide my students in developing some specific strategies to help them organize their thinking. Through a combination of modeling, providing opportunities for exploration, facilitating discussions about thinking, and prompting students to reflect on their experiences, I observed the continually increasing efficiency with which my students solved problems. The more they explored and analyzed problem-solving strategies, the more successful they became. Surprisingly, not just the most capable of my students showed progress, but all of them did. As I demonstrated various strategies to attack problems and began to let my students see math problems through visual and hands-on demonstrations, their skills improved. And my skills improved, too! The more comfortable I became at teaching problem solving, the more confident I became about my ability to help my students understand a process that had once seemed so complicated and abstract.

With an understanding of the problem-solving process and a repertoire of strategies to assist our students in dealing with problem situations, our anxiety and frustration lessens and our enthusiasm and confidence grow. Not all students can become effective problem solvers on their own, but with the help of a confident and capable teacher, all students can significantly improve their problem-solving abilities.

Developing Skills and Attitudes

Developing the problem-solving abilities of primary students is a challenging and complex task. It requires attention to the building of mathematical skills and thinking processes as well as attention to the development of positive attitudes toward problem solving. Both skills and attitudes must be strengthened to produce truly effective problem solvers.

Problem solving is a process that requires students to follow a series of steps to find a solution. Although some students may intuitively follow a process, many students need to be taught how to proceed to reach a solution. Another important goal in teaching students to solve problems is assisting them in developing strategies or plans for solving problems. Although choosing a mathematical operation (e.g., addition or subtraction) is frequently the way to solve a problem, alternate strategies are often needed. Helping students learn strategies such as drawing pictures, finding patterns, making tables, making lists, guessing and checking, working backward, or using logical reasoning gives students a wide variety of strategies to employ during problem solving. Problem solving requires this knowledge of strategies as well as the ability to determine when each strategy would be best used. The more our students practice these strategies, the more confident they become in their ability to solve problems and apply mathematics in meaningful ways.

The development of a positive attitude toward problem solving is crucial to student success. As teachers, we are instrumental in helping our students develop the attitudes needed to become successful problem solvers. Positive attitudes are built on an awareness of the nature of problem solving:

- *Problem solving requires patience.* It is not always possible to find a quick answer, and quick answers are often incorrect. Problem solving is not judged on speed but on the reasonableness of the final solution.

- *Problem solving requires persistence.* Students may need to try several strategies before finding one that will work. Students must have confidence that they can find a solution, even if it is not immediately apparent.

- *Problem solving requires risk taking.* Students need to be willing to try their "hunches," hoping that they may lead to a solution. Students must feel comfortable making mistakes, as problem solving is a process filled with mistakes that often lead to solutions.

- *Problem solving requires cooperation.* Students must often be willing to share ideas, build on one another's thoughts, and work together to find a solution.

Students become successful problem solvers when they are instructed in a climate that rewards patience, persistence, risk taking, and cooperation. As teachers, we have a critical role in establishing a positive climate for problem-solving instruction.

How This Book Will Help You

This book is designed to help you better understand the NCTM problem-solving standard. It explores problem solving as both a process through which students learn mathematics and a skill that enables them to apply the mathematics they have learned. The mathematical goals of students in prekindergarten through grade 2 are specifically addressed, and practical ideas are provided for helping primary students become effective problem solvers.

This book presents ideas for developing a problem-centered approach to teaching mathematics within your classroom. We will see how problem solving can set a context for learning math skills, can excite and engage students, and can help students discover insights and better understand math ideas. We explore ways in which problem solving enriches our math classrooms and nurtures enthusiasm, curiosity, and insight.

Within this book you will find a variety of ideas to help you better understand the problem-solving process, as well as specific strategies including Choose an Operation; Find a Pattern; Make a Table; Make an Organized List; Draw a Picture or Diagram; Guess, Check, and Revise; Use Logical Reasoning; and Work Backward. These strategies help students organize their thinking, figure out ways to approach and simplify problems, and ultimately find their way to solutions. We explore practi-

cal ways to support our students as they develop these thinking skills, knowing that the groundwork for each strategy is laid in the primary grades. As we investigate a variety of problem-solving strategies, we delve into their underlying skills in order to unearth the complexity and importance of each strategy. A variety of activities appropriate for primary students are shared for each strategy. Specific grade levels are not indicated on each activity, as problem-solving skills do not develop by grade level, but rather depend on students' prior knowledge and previous exposure to each strategy. Some primary students enter school intuitively demonstrating a strategy, while others may need exposure and repeated practice to develop the same thinking skill. Teacher tips are shared highlighting important points to emphasize when working with students. Examples of student work are presented for each strategy, including samples of students' communication about their problem solving. The student work samples offer a glimpse into students' thinking as their skills develop.

Once we have explored the problem-solving standard in depth, you will see how it connects to the math content standards in Chapter 13, titled *Problem Solving Across the Content Standards*. Through sample classroom activities, we explore the interconnectedness of the content and process standards. We discuss sample problem-solving tasks that blend with the math content taught in grades prekindergarten through 2 in numbers and operations, algebra, geometry, measurement, and data and probability. Student work is shared to illustrate these lessons, and you will be asked to reflect on the combined teaching of math content and the problem-solving process.

In this book we also discuss the assessment of problem solving, including the use of observations, interviews, and rubrics to assess students' skills. While this book is designed to help you better understand the NCTM Problem-Solving Standard and to provide you with practical ideas and classroom activities related to the standard, it is also intended to stimulate thought about teaching and learning. Following each chapter, several questions prompt you to reflect on the content of the chapter whether alone or with a group of your colleagues. Taking a moment to reflect on the ideas presented and to relate them to your teaching experiences and observations of your students will help you better process the ideas and apply them to your students' specific needs.

A very important component of this book is the inclusion of the practical resources needed to implement the ideas explored throughout the chapters. The accompanying CD is filled with a variety of teacher-ready materials to help you implement a problem-solving program in your school or classroom. Graphic organizers, evaluation forms, a scoring rubric, and icons are all available, as well as a variety of practice problems for your students. The practice problems range from simple to complex. Some of the CD activities appear as teacher notes, providing you with directions for conducting the activities in your classroom. Other activities are provided in worksheet format for those students who may be able to work independently. If your students are unable to independently read the directions, however, these activities can be modified through teacher-read directions or simply be used to generate ideas for similar problem-solving activities or classroom demonstration. Select those activities that suit your students' level of expertise, and continue to challenge your students with more sophisticated problem-solving tasks as their skills improve. And the activities and resources on the

CD can be easily modified to suit your students' specific needs. Change the data to make an activity less or more challenging, or insert familiar names and places to engage and motivate your students.

It is hoped that this book will enhance your understanding of the problem-solving standard and provide you with insights, resources, and practical ideas to develop your students' problem-solving skills. When we, as teachers, better understand the complexity and importance of problem solving, we are better able to identify, select, and design meaningful tasks for our students. It is hoped that the varied instructional practices highlighted in this book will assist you in developing your students' skills and expanding your own understandings. Most certainly, as we reflect on and develop our teaching skills, our students' problem-solving skills will increase as well.

Questions for Discussion

1. Were you taught how to solve math problems or just assigned problems to solve? How did you feel about math problem solving when you were a student in the math classroom? In what ways do your past experiences and attitudes about problem solving influence your teaching of problem solving?

2. If students show competence with computational skills but lack problem-solving skills, how might it affect their math achievement? What possible problems might they experience?

3. What attitudes are essential to be an effective problem solver? How might you support students in developing these attitudes?

4. What skills are essential to be an effective problem solver? How might you help your students to acquire those skills?

Bear Watching

The Task:

Students will explore addition problems using a story mat and stories about watching bears at the zoo.

Directions:

1. Provide each pair of students with a story mat and talk about the parts of the mat. Ask students if they have ever been to a zoo. Tell them that the bears in a zoo might like to swim in a cool pond, or sun themselves on a big rock, or climb in a tree, or even sleep under a shady tree. Have students locate those places on their story mats. If you'd like, have students color the water blue, the rock brown, and the tree green.
2. Provide students with some bear cutouts or other manipulatives to represent bears. Tell the students that they will be solving math problems about bears at the zoo. Begin by posing some simple tasks:
   - Put 2 bears on the rock.
   - Put 4 bears in the pond.
   - Put 3 bears sleeping under the tree.
   - Put 1 bear climbing in the tree.
3. Ask students to clear their mats, then pose an addition scenario and ask them to place bears on their mats to show the story. After each scenario, have students clear their mats and then pose another one. Some examples:
   - 3 bears were in the pond. 2 bears were on the rock. How many bears were there altogether?
   - 2 bears were sleeping under the tree. 3 bears were climbing in the tree. How many bears were there?
4. Have students talk with partners to solve the problems. Then ask them to share their answer with the class and ask them to explain how they solved the problems (e.g., counting both groups of bears together).
5. Record the addition number sentence on the board after solving each problem (e.g., 3 + 2 = 5). Talk about the concept of addition as putting things together or joining groups. Frequently remind students that they were adding (joining) the groups of bears.

6. Have students work with their partners to create an addition story with their bears. Have them tell you the story and the number sentence to match their story. As students share, focus on groups being joined in their stories.

Materials:

For each pair of students:

- Story mat
- 10 bear cutouts or other manipulatives to represent bears

Talk About It:

Try these questions to stimulate thinking and discussion:

- How did you get the answer?
- Why did you count the bears in the pond *and* on the rock?
- How could you write that in a number sentence?
- Can you come up with a story in which we add bears?
- Why did we add the bears in your story?

Write About It:

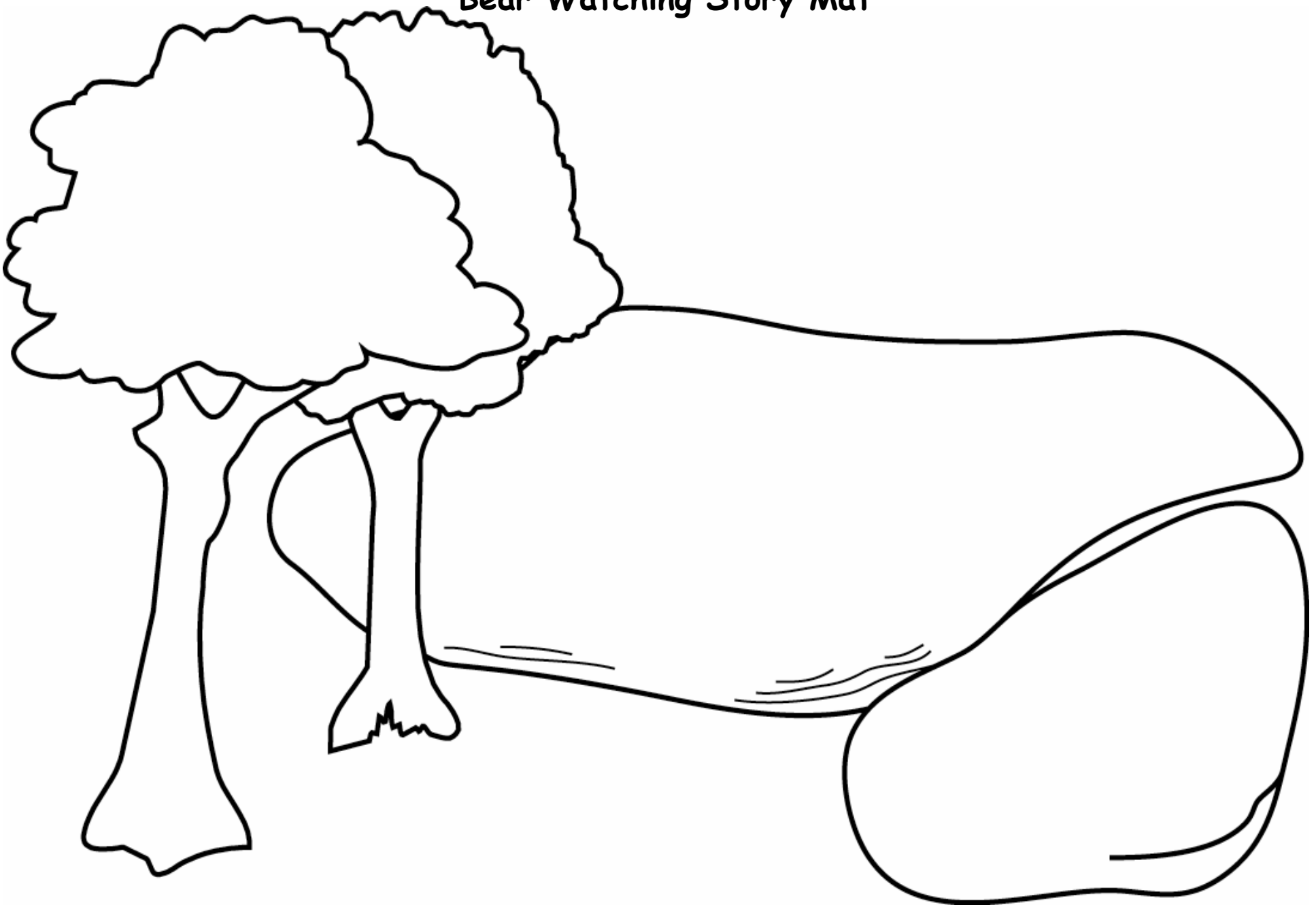
- When you are finished posing various scenarios, students might be asked to create a story, glue bears to their mat to show their story, dictate their story to you, and then record the number sentence to show the addition problem. For students who are able to do so, have them write their stories on the back of the mat.

Tiered Learning:

Adjust or extend the activity in the following ways:

- Do the activity as a demonstration, making an overhead of the story mat. Have students interact with you as you pose problems and talk about how to solve each one.
- Pose subtraction stories using the bears and mat (i.e., 5 bears were in the pond. 3 bears got out and sat on the rock. How many bears were left in the pond?)

Bear Watching Story Mat



Bear Templates