Introduction to Connections

Grades PreK–2

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The Math Process Standards Series
Susan O'Connell, Series Editor

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CONTENTS

FOREWORD

In order to be effective mathematicians, students need to develop understanding of critical math content. They need to understand number and operations, algebra, measurement, geometry, and data analysis and probability. Through continued study of these content domains, students gain a comprehensive understanding of mathematics as a subject with varied and interconnected concepts. As math teachers, we attempt to provide students with exposure to, exploration in, and reflection about the many skills and concepts that make up the study of mathematics.

Even with a deep understanding of math content, however, students may lack important skills that can assist them in their development as effective mathematicians. Along with content knowledge, students need an understanding of the processes used by mathematicians. They must learn to problem solve, communicate their ideas, reason through math situations, prove their conjectures, make connections between and among math concepts, and represent their mathematical thinking. Development of content alone does not provide students with the means to explore, express, or apply that content. As we strive to develop effective mathematicians, we are challenged to develop both students' content understanding and process skills.

The National Council of Teachers of Mathematics (2000) has outlined critical content and process standards in its *Principles and Standards for School Mathematics* document. These standards have become the roadmap for the development of textbooks, curriculum materials, and student assessments. These standards have provided a framework for thinking about what needs to be taught in math classrooms and how various skills and concepts can be blended together to create a seamless math curriculum. The first five standards outline content standards and expectations related to number and operations, algebra, geometry, measurement, and data analysis and probability. The second five standards outline the process goals of problem solving, reasoning and proof, communication, connections, and representations. A strong understanding of these standards empowers teachers to identify and select activities within their curricula to produce powerful learning. The standards provide a vision for what teachers hope their students will achieve.

This book is a part of a vital series designed to assist teachers in understanding the NCTM Process Standards and the ways in which they impact and guide student learning. An additional goal of this series is to provide practical ideas to support teachers as they ensure that the acquisition of process skills has a critical place in their math instruction. Through this series, teachers will gain an understanding of each process standard as well as gather ideas for bringing that standard to life within their math classrooms. It offers practical ideas for lesson development, implementation, and assessment that work with any curriculum. Each book in the series focuses on a critical process skill in a highlighted grade band and all books are designed to encourage reflection about teaching and learning. The series also highlights the interconnected nature of the process and content standards by showing correlations between them and showcasing activities that address multiple standards.

Students who develop an understanding of content skills and cultivate the process skills that allow them to apply that content understanding become effective mathematicians. Our goal as teachers is to support and guide students as they develop both their content knowledge and their process skills, so they are able to continue to expand and refine their understanding of mathematics. This series is a guide for math educators who aspire to teach students more than math content. It is a guide to assist teachers in understanding and teaching the critical processes through which students learn and make sense of mathematics.

Susan O'Connell
Series Editor

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The Connections Standard

> *Through instruction that emphasizes the interrelatedness of mathematical ideas, students not only learn mathematics, they also learn about the utility of mathematics.*
>
> —National Council of Teachers of Mathematics, *Principles and Standards of School Mathematics*

Why Focus on Connections?

By the time they reach first grade, many students already view mathematics as a collection of isolated skills and concepts that they must work on, in school, during mathematics class. Although they may not know what they will be studying when the school year begins, they quickly learn (as daily objectives are written on the board and announced prior to the beginning of mathematics class) that they are expected to learn these concepts to move on to the next grade. Textbooks, which provide teachers with a guide for instruction, often include a week of study on some skill, only to move on, in the next chapter, to some new skill unrelated to what was just introduced. A week of addition may be followed by several days exploring the attributes of plane figures and then proceed to ideas about fractions. This may cause some students to see mathematics as a fragmented, linear progression of skills like an unassembled puzzle. A goal of mathematics education "is to present mathematics as a unified discipline, a woven fabric rather than a patchwork of discrete topics" (NCTM 1995, vii). Imagine how empowered students would be if they could begin linking the pieces of the puzzle together to reveal a more focused picture of mathematics.

In this picture, knowledge of the part-part-total nature of numbers and shapes could be connected to basic addition facts and decomposing plane figures into other plane figures. Once students understand that all numbers can be taken apart and put back together (in different ways), they can use this to help them understand our place value system. The traditional algorithms for addition and subtraction of multidigit numbers would make more sense if students understood that five tens and three ones was just one way to think about fifty-three. If they knew that this number could also be named as four tens and thirteen ones (or three tens and twenty-three ones), they would more easily see how the traditional renaming procedure for subtraction "works."

The National Council of Teachers of Mathematics states, "These connections help students see mathematics as a unified body of knowledge rather than a set of complex and disjoint concepts, procedures, and processes" (NCTM 2000, 200). One might question whether prekindergarten through second-grade students ever think about or even care about such things—and the answer is probably "No." What does happen is that these young students begin to realize that the mathematics they're learning in school is unrelated to their lives. When this happens year after year, many students consider that mathematics is lacking in value and utility. And they may stop caring about learning it.

Students who fail to recognize connections are not necessarily unsuccessful in mathematics. Many are able to remember skills and procedures and perform well on various assessments. However, that success is often isolated to specific skills and concepts used in specific situations and, in many cases, may be short-lived. Our role as educators, therefore, is to be mindful of the multifaceted nature of mathematics and to bring such connections to light in the classroom.

There are numerous benefits for students who recognize connections among mathematical ideas, between mathematics and other disciplines, and in life experiences. When students understand the interrelatedness of mathematics, they often have many more strategies available to them when solving problems, and more insight into mathematical relationships (Cobb et al. 1991). These students often develop their own procedures, based on an understanding of place value ideas, rather than mimic a particular strategy or algorithm to reach a solution. Additionally, when students construct knowledge and form connections, they are more likely to transfer conceptual knowledge and apply it to new situations. A deeper level of understanding equates to greater utility and versatility of the knowledge by the learner. The more connections students are able to recognize, the deeper the level of sense making. It is "when students can connect mathematical ideas, [that] their understanding is deeper and more lasting" (NCTM 2000, 64). These connections contribute to a strong and cohesive foundation of knowledge, a fundamental necessity on which to build future knowledge and lifelong understanding.

Children often create connections on their own based on their real-life experiences. A Family Circus cartoon (Keane 1994) shows a young boy looking at an analog clock that has the hour hand near the three and the minute hand on the ten. The child says, "The big hand is on channel 10, and the little hand is on channel 3." Although sweet and somewhat funny, the implication is that children look for ways to connect what they know with new things that they are learning.

At the conclusion of a lesson in a first-grade classroom on a Friday afternoon, the following question was posed, "How might you or your family use mathematics over the weekend?" After a considerable amount of prodding and waiting, these responses were given:

"We don't get weekend homework, but if we did, I'd use math to do my homework."
"My mom could help me with my math homework, if we had some to do."
"Maybe I'd see math on *Sesame Street,* like counting."
"I could count the steps to my room."

That was it! This revealed that these students neither recognized the utility of the mathematics lessons just completed, nor the application of mathematics to their own lives. It seemed that they just believed that mathematics was a subject they learned in school. It seemed that unless mathematics was connected to school (through homework or some other means of practice), they wouldn't be doing mathematics over their weekend and neither would anyone in their family. As teachers, we need to work hard to provide opportunities for students to recognize and celebrate the connections within mathematics and to their lives—now, and in the future. Many of these efforts are shared with you in this book.

What Is the Connection Process Standard?

The National Council of Teachers of Mathematics (NCTM) has developed standards to support and guide teachers' planning for mathematics instruction. These standards include guidelines for instruction in both content and process. The content standards define specific topics of mathematics, and the process standards identify the modes by which students engage in mathematics. The process standards include problem solving, reasoning and proof, communication, connections, and representation. The components of the NCTM Connections Process Standard, the focus of this book, are described by the following expectations (NCTM 2000, 64):

Instructional programs should enable students to—

- recognize and use connections among mathematical ideas.
- understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
- recognize and apply mathematics in contexts outside of mathematics.

This book explores strategies, activities, and materials designed to assist students in developing a more comprehensive understanding of mathematics by focusing on the connected nature of the subject. Each of the three focuses within connections is covered, and a multitude of resources for classroom use are offered.

The first focus is on the connections that exist among the various content areas that teachers introduce and reinforce during mathematics instruction. Seldom, even in a textbook lesson, are skills taught in isolation from other mathematics topics being learned or reviewed. This is especially true when "rich tasks" and interesting problem-based situations are given to students. One need only look at a "typical" logic problem to see how easy it can be to connect mathematics ideas. (See What Number Am I? in the Mathematical Ideas Interconnect and Build Upon One Another section of the CD.)

What Number Am I?

I have exactly two digits.
The digits are different.
I am an even number.
You say this number when you count by tens, starting with ten.
I am greater than 50.
I am less than 80.
A quarter and a nickel combined is half of me.
What Number Am I? ____ ____

Did you follow the clues to solve the puzzle? Did you get "60"? And did you see all of the mathematics that a student would get practice with in solving a problem like this one?

Many important vocabulary words and mathematics concepts are reinforced in the context of this problem (*digit, even, exactly, greater than, less than, the value of a quarter and a nickel, half, what numbers are said when counting by tens starting with ten*). Knowing the meaning of these terms is important, but equally important is knowing what to do with these words. If the term *combined* is used, students must know that they will be adding, and they need a strategy for getting the sum. Just knowing the phrase *counting by tens* doesn't help a student unless the student has memorized the sequence of number words said when you count by tens, beginning with ten. This particular logic problem also requires an understanding of the relationship that exists between numbers, an area of number sense that should be reinforced often with primary students. All of these clues (with the exception of the "money" clue) fall within NCTM's Number and Operation content standard (NCTM 2000). When a teacher makes connections among mathematics concepts and skills, many ideas can be reinforced at the same time.

The second focus is on the connections that exist within the numerous skills and concepts within mathematics itself, and how mathematical ideas interconnect and build on one another. One example of such a connection would be helping students recognize that "5" is the same as a "1" and a "4" on a number balance, a "2 + 3", and a "6 – 1." These types of connections require an understanding that mathematical ideas interconnect and build upon one another to produce a coherent field of study.

In geometry, for example, early identification of triangles, squares, and rectangles enables students to makes sense out of ideas of symmetry and congruence in later grades. Knowing this allows learners to better understand what happens to these shapes when they are rotated, translated, and reflected. For these things to happen, teachers must understand how essential these prerequisite understandings are for later mathematical understandings. But it is also critical for students to make these connections to fully comprehend how earlier understandings make sense when new concepts are presented.

A third focus is to explore the connection between mathematics and other areas of the elementary curriculum. In his chapter on connecting literature and mathematics, David Whitin writes, "an effective strategy for restoring context to mathematical ideas is through the use of children's literature" (NCTM 1995, 134). Today a multitude of math-related literature exists, with many books specifically designed for use with primary students. Interdisciplinary approaches to teaching, such as using the mathematics–literature connection, save precious time and also add to students' insight into all curriculum areas involved, thereby reinforcing that "The whole is greater than the sum of the parts" (Welchman-Tischler 1992, 1). Literature may provide the catalyst for problem solving, but having students write to explain the strategies they've used to solve these problems can solidify and help them share their thinking.

Spatial awareness and spatial concepts can be integrated into physical education programs, patterns that repeat and the study of plane and solid figures can be studied in art, and rhythmic patterns and an introduction to fractions can be reinforced during music. Even science and social studies offer a multitude of opportunities to make connections to a variety of mathematics ideas.

While still a part of the connections to other areas of study, a final connection will be made between mathematics and the real world. When money is studied without relating it to getting an allowance, going shopping, and saving to purchase something special, sense-making opportunities are lost. Students need to wonder why rectangles can be seen in most rooms but that triangles aren't found nearly as often. They also need to see the connection between learning to tell time on an analog and digital clock and why it's necessary to do this in the first place. These are questions and ideas that students need to explore to appreciate the applicability of mathematics to their world. Such an understanding can serve as a motivating factor for many young learners, as it provides a rationale for engaging in mathematical explorations.

Developing Skills and Attitudes

Recognizing connections in mathematics requires that students consider this content in a new way. Not only must they grasp the necessity of the skill within a particular lesson, but they must also reflect on how this knowledge might relate to past understandings and future experiences. Students are required to think beyond one lesson, one concept, and one application in mathematics. Seeking connections must become a habit of the mind for students. This process is not instinctive for many students; rather, it is learned. As their teachers, it is our responsibility to model these behaviors for students and provide prompts that promote such behaviors. Questioning is one method of promoting this process. We may pose such questions as:

How does this relate to yesterday's lesson?
Would the strategy you developed work with this problem?

When might this be applicable outside of math class?
Who might need to know this information?

Our goal in posing such questions is to model for students the types of questions they should be asking of one another and of themselves. Students then become involved in the process of building knowledge and making connections, and thus become more accountable for their learning.

It then becomes our job to facilitate a learning environment in which students feel comfortable engaging in discourse to reveal such connections. We want students to rely on themselves and one another to unveil the interrelationships that exist in and between mathematics throughout the curriculum, and in their real world. This occurs in a classroom where students are engaged and invested in the lesson.

CLASSROOM-TESTED TIP

Think-Pair-Share, a cooperative learning strategy, provides students with time to reflect on a question or problem being asked, without dealing with hands being raised and answers being blurted out. We ask students to silently think (for about ten to fifteen seconds) when a question is asked. Then we have them pair up with the person next to them or across from them to quietly share their thoughts. This allows many students to talk at the same time, and gives those who may not have had an idea about an answer an opportunity to hear what their partner thinks. Then, after about a minute of pairing, students are asked to share (out loud) the things that they've heard or the things they had been thinking about. Students are more invested in the lesson, and they are more willing to participate when this strategy is employed.

Additionally, we must choose tasks that are meaningful and accentuate the connections among content areas and extend to other disciplines. This often means that a few activities are explored in depth. Lastly, we must be knowledgeable about students' prior knowledge and know the content and skills to be taught in later grades. We cannot limit our expertise to the particular grade level we teach. Only then may we empower learners to develop a strong, well-connected knowledge base. "Building on connections can make mathematics a challenging, engaging, and exciting domain of study" (NCTM 2000, 205).

The question, "How will you use mathematics over the weekend?" will still be asked of students. Because homework is seldom given to young learners, we might ask instead, "When will you be using mathematics once you leave school today?" Students may still respond that "I'll be doing my math homework over the weekend, or once I'm home." But we also expect many students to make other statements that let us know they understand the value of the mathematics they are learning. They might say, "I'll have to know when to come in for lunch. So I'll look at my watch and know what time it shows."

"I'll set the table for Sunday dinner and will have to count out the forks, knives, plates, and glasses."

"As soon as I get up on Saturday, I'll be looking at the clock to see what time it is."

"All you have to do is look around to see that math is everywhere. It's the shapes in buildings, the numbers on speed signs, and in so many things that I do!"

How This Book Will Help You

This book is designed to help you better understand the NCTM process standard of connections. It explores ways to help students make sense out of the connections between mathematics ideas, how these ideas build on one another, and how mathematics can be applied in contexts outside of the mathematics classroom. This book focuses specifically on the mathematical expectations of students in prekindergarten through grade 2, and it provides practical ideas for helping students recognize, seek, and apply connections. It offers ideas for creating a classroom environment that acknowledges the numerous connections inherent in the field of mathematics, as well as among mathematics and other areas of the curriculum.

Specific grade levels are not necessarily indicated for each activity. Activities are provided to introduce and reinforce important teacher strategies necessary in creating a learning environment that encourages and applauds the recognition of connections.

Classroom activities are provided to explore the relationship of the content standards and the connections process standard. Examples of student work are included to more clearly illustrate students' understandings based on their ability to make connections. Often these student work samples illustrate their attempt at making sense of the mathematics they've learned in the past and the mathematics being introduced to them presently. Their justification is often a means to explain a particular strategy's effectiveness in reaching a solution, which, in turn, opens the door to a variety of connections within mathematics.

The final chapter explores techniques in measuring students' abilities to make connections. Assessment samples take the form of classroom discourse, written formative assessments, and individual conferencing with students. Student responses are provided, as well as conclusions summarizing student knowledge and potential gaps revealed through work samples. Additionally, recommendations for instruction based on the examination of student work serve as suggestions for future instruction.

The accompanying CD includes a variety of teacher-ready materials to aid in the recognition of connections in the mathematics classroom. Some of the CD activities appear as teacher notes, providing you with directions for conducting the activities in your classroom. Other activities appear in worksheet format, for those students who may be able to work independently. If your students are unable to independently read the directions, however, these activities can be modified through teacher-read directions or might simply be used to generate ideas for similar problem-solving activities or classroom demonstration.

Lastly, each chapter concludes with questions for discussion. These prompt you to reflect on the content of the chapter, either individually or with a group of colleagues. They offer you an opportunity to engage in discussions about the appropriateness of what has been read and the impact this might have on student understanding. Practical resources are listed to facilitate the implementation of ideas explored throughout the chapters.

The consistent practice of making and applying connections in mathematics contributes to students having a deeper level of mathematical understanding. As educators, we must create an environment in which students are taught to seek out such connections, and thus think like mathematicians, celebrating the relevance and applicability of mathematics.

Questions for Discussion

1. How were connections highlighted and emphasized in the mathematics instruction that you received as a young learner?

2. What learning experiences did you have that helped you make a connection among mathematical ideas, or that strengthened your understanding of a particular concept?

3. If students show competence with computational skills but fail to recognize the connectedness and utility of mathematics, how might this affect their achievement?

4. What changes might you make in your planning or instructional practices to facilitate student recognition of connections? What would you need to do to make these changes a reality?

Bean Out Directions for the Teacher

The Task

Either working independently or with a partner, students will match a numeral with a number, a number with a number, a numeral with a number word, or a numeral with the tally marks that represent this amount.

Directions

1. This early learning activity is meant to give students practice with identifying (recognizing) numerals, numbers, number words, and tally marks as representations of quantities. Students will be given a sheet with one of these representations on it. Then they toss either a decahedron die or a hexahedron die (number/numeral cube) and place a bean over the top of the number or numeral that matches.
2. If this is an independent activity, model what students are to do by tossing the number generator, identifying what is on the die, and placing a bean somewhere on the game board. Students play the game until all of their numbers or numerals are covered by beans.
3. If this is a paired activity, students can use bi-colored beans or bi-colored discs. They take turns with the number generators and see who has the most of their color once every numeral or number has been covered. If a student tosses the number generator and cannot cover a place on the game board, the student loses his or her turn and gives the number generator to his or her partner.
4. This can also be a teacher-directed activity with a small group of students who may need additional supervision or assistance.

Materials

- Bean Out game board (either numerals, numbers, tally marks, or number words)
- Kidney beans (if played independently)
- Bi-colored beans or discs (if played with a partner)
- Number or numeral die (hexahedron or decahedron)
- Cup with a lid to put the die inside (so it doesn't fall to the floor)

Talk About It

- Before playing the game, ask students to look at the numeral/number generators and think of things to say about these.
- Have them do the same thing with the Bean Out game board.
- Ask students whether they see the same numerals/numbers more than one time.
- Ask them how many total places there are on the game board.
- Ask them to talk about how they will take turns with their partner (if they are playing this as pairs)

Write About It

- Because this game is geared more toward prekindergarten and kindergarten students, it is unlikely that students will be writing about this activity.
- Instead of a written exercise, ask students to create a "Learning Experience Story" to share how they liked playing the game and what they learned while they played it.

Tiered Learning

- This activity is differentiated by virtue of the fact that the different game boards work on different levels of understanding. If you have students who just need to match a numeral with a numeral or a number with a number, give them the game board that has the same things on it as the dice have on them. If the die has "pip" arrangements, the game board should also have "pip" arrangements. This is also true if you want students to match a numeral with another numeral.
- At the second level of difficulty, students will match the numeral with a number. So the die could have "pips" and the game board could have numerals.
- At the third level of difficulty, students match numerals or "pips" with number words.
- For a fourth level of difficulty, use a decahedron die and ask students to match the numeral on the die with the tally mark arrangements.

Bean Out Folder Game Directions

Materials

Folder with Bean Out game board
Either one or two numeral or number cubes (depending on level of difficulty)
Some sort of bean to use as a counter
Small cup to put beans into

Directions

This game can be played either independently or as a paired activity. The student tosses the number or numeral cube and finds its match on the Bean Out game board that is being used. Play continues until all of the spaces have a bean on them. If attention level is not high, continue playing only until one player has five in a row (diagonal, horizontal, or vertical).

Name ___________________________________

Bean Out

3	8	9	7	5
6	11	7	5	4
10	7	3	8	5
12	4	9	5	6
3	2	10	7	8

Name ____

Bean Out

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Name ________________________________

Bean Out
