



**preK-grade2**

# Differentiating

**in Number & Operations**

and the Other Math Content Standards

- A Guide for**
-  Ongoing Assessment
  -  Grouping Students
  -  Targeting Instruction

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When your heart is in your work, it doesn't seem like work. That doesn't mean it is effortless. It just means you enjoy it.

This book is dedicated to all of the students, teachers, administrators, and parents that I have worked with over the years. It is for you and because of you that this book was written.



## Why Do We “Aim for the Middle” When We Teach Math?

*Aim for the middle and pray for ricochet!*

Let’s face it. Most math lessons are directed toward the middle of the group. We ignore the reality that some students already know the information and some students are not ready to learn the information. We aim for the middle and pray for ricochet. We hope the knowledge we impart to the center will bounce around until everyone gets it. I used to teach this way. Many educators teach this way. Why?

Perhaps we aim for the middle in our instruction because we are inundated with standards-based ideals. Politically charged and socially propelled standards drive educators to teach the curriculum. The merits of standards-based instruction and increased expectations are sound, but assuming that all students at the same grade level need to learn the same thing at the same time is a problem.

Our national focus on assessment in mathematics may also encourage a tendency to teach to the middle. However, it is important to remember that educators need to incorporate mathematics assessment in ways that are beneficial to both students and teachers, and that not all assessment involves students taking state and national tests. Ongoing assessment tells us where students are in their math knowledge before, during, and after instruction. It is the heart of differentiated instruction in math. Educators

can use formal, informal, and personal types of assessment. Such tools provide opportunities to question learners in ways that reveal their thinking and provide insight to the teacher. As we implement productive assessments, we gain rich information and come to understand and value assessment as a critical tool for differentiated instruction. Perhaps another reason we aim for the middle is that we simply do not have command of the content. Teachers need to pay attention not only to concepts themselves but also to what comes before and what comes after those concepts. Let's say my goal is to teach addition using single-digit numbers to a class of first graders. Some of my students may already be comfortable working with two-digit addends, while others may not yet understand what addition is. Often, teachers are left with no clear guidance as to what to do for these students, and the aim for the middle approach isn't going to help them.

Perhaps it is an issue of pedagogical expertise. Think about how many times you have heard a teacher say, "I taught it, but they didn't learn it!" What the teacher may actually be saying is, "I told them or I showed them, but they didn't get it." Traditional mathematics instruction often includes a lecture followed by examples shown to the whole class. Unfortunately, this type of instruction is not suited for all students. Still, in many classrooms, the instruction centers around the teacher demonstrating problems and concepts on the board or overhead projector. The teacher, then, often is the only person actively engaging in mathematics.

Recent professional development opportunities for math teachers have started to address some of the pedagogical and contextual hurdles. With an increased focus on active learning, many math classrooms—especially grades P-2 classrooms—have come alive with hands-on mathematics. Teachers are beginning to express more comfort with mathematics. However, the one-size-fits-all whole-group approach remains pervasive. Consequently, we still have students frustrated because the level of instruction is beyond them; we still have students bored because they already know the information. All students are entitled to instruction that meets their needs, not just those fortunate enough to be in the middle.

The answer is clear—targeted, differentiated instruction; yet it is hard to find truly differentiated math classrooms. Teachers tell me, "Of course, I want to meet the needs of all my students in mathematics, but I don't have time." "Yes, I believe in the power of differentiated instruction, but not in math. We have too much to cover." "Differentiated instruction is great for literacy and social studies, but how to use it in math is a real mystery." These statements and others launched me into a quest for making differentiated instruction in mathematics a reality. My journey started

in my classroom and has expanded through professional development models to classrooms across the country. While there are numerous ways to differentiate instruction, this book focuses on a targeted approach to differentiation of math content. This focus is in no way intended to minimize the positive impact associated with other types of differentiation. In fact, we'll explore the importance of incorporating students' learning frameworks which include styles, preferences, needs, and interests, because these are powerful ways to increase student achievement.

The problem is that there are so many avenues and so many possibilities when differentiating instruction that implementing it can become an unobtainable ideal instead of a reality for many educators. To remedy this problem, I encourage teachers to begin by focusing on one aspect of differentiated instruction—not at the expense of other ways to differentiate, but as a way to start. Given today's focus on standards-based instruction, I prefer to begin with the math content. My goals are to use quick and effective ways that are research-based and have been tested in hundreds of classrooms to assess student understanding; employ purposeful, flexible grouping; utilize levels of cognitive demand; and target instruction for all students. In order to differentiate instruction in math using a standards-based curriculum, without extra hours of planning, I target math instruction for students. You can easily implement these strategies in your own classrooms.

## **Why Is the Focus of This Book on Number and Operations?**

*Why is number so important?*

The number and operations content strand provides a foundation for all other areas of mathematics. All students need to establish solid groundwork in number and operations to help them be successful in mathematics. The goals are to help students foster their number sense by working with numbers in accurate, efficient, and flexible ways. To do so students need their teachers to provide targeted, differentiated instruction in number and operations.

For young students, number and operations involve much more than counting or writing numbers. Math educators need to understand all of the components that comprise number and operations so that they can easily target the instruction for students.

## Using This Book with the Content Companions

### *Why are there Content Companions?*

The purpose of the Content Companions is to provide prekindergarten through second grade mathematics educators with the tools needed to target instruction through differentiation in all math content areas. The number and operations book offers ideas, techniques, and strategies to use when targeting instruction in number and operations. The Content Companions allow educators to understand how to use these same ideas, techniques, and strategies when teaching algebra, geometry, measurement, and data analysis and probability. The Content Companions offer specific content examination, student work samples, and lesson ideas. The rationale behind the Content Companions is that while number and operations is a critically important math standard, it is not the only standard that prekindergarten through second grade math educators teach. Algebra, geometry, measurement, and data analysis and probability are also very important content standards.

Throughout this book and the others in the set you will notice margin boxes that indicate where other mathematics content explanations and examples are offered in the Content Companions. Each time you see the cross-reference box in the margin in any of the books, you can turn to the corresponding page in this book or the Content Companion to find specific information pertaining to number and operations, algebra, geometry, measurement, or data analysis and probability. There are far too many books out there that attempt to explain to readers that similar strategies can be applied in all areas of mathematics; but these resources often fail to provide specific examples. Likewise, some resources attempt to cram all content into one resource; but these resources are often limited and incomplete. To address these problems, the Content Companions provide a comprehensive set of resources that prekindergarten through second grade educators can use to target instruction through differentiation in mathematics.

The fabulous news is that we don't have to teach to the middle and pray for ricochet anymore! We have the knowledge and tools to help us meet the needs of young math students. I invite you to read these books to learn how to implement a targeted approach to differentiated instruction in mathematics. Please accept my invitation. . . .

# Targeted, Differentiated Instruction

## What Is a Targeted Approach to Differentiated Instruction in Math?

### *Targeted instruction in action*

Classroom teachers are busy with no time to reinvent the wheel again and again. We need quick and effective ways to target math instruction. The beauty of this book is in the real classroom examples and explanations. Without lengthy lesson plans or hours of preparation, teachers can easily implement targeted, differentiated mathematics instruction on a daily basis. By knowing where students are in their learning journeys and connecting this knowledge to curricular goals and standards, differentiation in mathematics can become as second nature as “hands-on learning.”

### **Command of the Content**

Targeting math instruction on the fly requires us to encompass a new way of thinking about teaching. What to teach and how to teach are important, but they are not enough. Simply put, even the best-scripted lessons will never accomplish what targeted, differentiated instruction can accomplish. We need to understand mathematics in a way that allows us to identify the previous concepts (back-mapping) and subsequent concepts (forward-mapping). The great news is that many districts across this country have already organized this information for us! Math researchers and math

supervisors have created various models of the goals and objectives for each grade level. However, when teaching first grade number and operations, the teacher needs to know the number and operations concepts that are taught in kindergarten and in second grade. The teacher may even need to know the prekindergarten and third grade objectives. This knowledge enables the teacher to have command of the content—allowing the teacher easily to build bridges, repair gaps, and provide challenges for all students.

I know what you are thinking . . . teachers have enough with their own grade level benchmarks. However, if we focus only on one grade level, we may not reach many students, because students are at various levels of understanding. “When a teacher tries to teach something to the entire class at the same time, chances are, one-third of the kids already know it; one-third will get it; and the remaining third won’t” (Lilian Katz, in Tomlinson 1996). Essentially, two-thirds of the students are not learning anything. To meet the needs of all students, try looking at one topic at a time. Find out the benchmarks for the previous grade level and find out the benchmarks for the next grade level. You’ll then have valuable tools for targeting math instruction, because you will have a greater understanding of the sequence of skills within any given concept, allowing you to use back-mapping and forward-mapping. Having command of the content enables the teacher to target instruction on the fly.

Traditionally teachers thought—and were often told—that they could not tread on another teacher’s turf. If you teach something that the student is supposed to learn next year, what will the teacher teach next year? I remember being told by a supervisor many years ago, “Don’t even look at next year’s math objectives because you don’t want to accidentally teach something that is supposed to be learned in the next grade.” How disconcerting, perhaps even scary, this advice is. Let’s move past this type of thinking. Every student deserves to have learning opportunities targeted to specific academic needs. If next year’s teacher is good, she will continue to differentiate instruction and will have plenty to do! Even if she does not differentiate instruction, you will have provided optimal academic growth, ultimately better preparing the students for their next classroom. You may have even inspired many of the students to seek out self-directed opportunities to continue learning at their levels.

### **Time Is Precious**

Time is so valuable. We always need more of it. There’s simply not enough time. Not enough time to teach. Not enough time to plan. Not enough time to assess knowledge. Not enough time to reflect upon our practice, let alone change the way we are teaching. Perceived lack of time often keeps teachers from trying new strategies or implementing new programs. The brilliant thing about tar-

geted, differentiated instruction, however, is that it can actually give you more time in the classroom. The more you differentiate instruction, the more time you actually have to focus on essential instruction. The teacher is not teaching the curriculum; instead, the teacher is teaching children. It doesn't happen overnight. Targeted, differentiated instruction is just like most everything else—the more you learn about it and practice it, the better you become at it.

Think about the first lesson plan you ever wrote. If you are like most novice educators, you devoted more time to thinking about and writing the lesson than you actually did teaching the lesson. Now, as a seasoned educator, you don't write lesson plans in the same manner. Sure, you may write an extensive lesson for an observation or for course work, but you no longer write lengthy, detailed lesson plans on a daily basis. You went through the process of learning how to teach and now you know what to do. The process for learning how to target differentiated mathematics instruction is similar.

To target instruction in mathematics without adding extra planning hours to my day, I think in threes—concept, next concept, and preceding concept. I define the math concept and think about how to bump it up to a higher level and how to scaffold it down to a lower level. Thinking in threes prepares me for the potential levels of my students. My goal is to have ideas to challenge those who already know the concept and to build bridges for those who are not yet ready to learn the concept. The challenging and the scaffolding are manifested through the adjustment of the levels of cognitive demand and targeted instruction, both of which we will examine in detail in this book.

Successful teachers are time grubbers. Because there is rarely extra time, teachers need to look for small chunks of potentially available time during the course of the school day. Analyze your day. Look for idle time. Can I meet with a small group during the morning routine? Can I assess learning during centers or seat-work? Is it possible to have a five minute reteach group while students pack up supplies at the end of the day? Can I teach students how to lead a portion of circle time (time where the class sits as a whole group to discuss various daily routines such as calendar, weather, announcements, daily schedule, and so on) so I can explain the math challenge to a group of students? Find the hidden time and use it to target and differentiate instruction. Every minute counts!

## **Leading and Managing the Class**

Contrary to popular belief, a differentiated classroom necessitates more leadership than a nondifferentiated classroom. As Tomlinson (1995) clarifies, teachers who differentiate instruction “exert more control in their classroom,

not less” (5). We need to identify our definitions of leadership and control. They do not mean dictatorship. The leader is not the constant sage on the stage. Controlling the classroom does not mean students must remain silent during math time. Instead, the teacher provides vision, support, and challenge to facilitate student learning.

In addition to being a strong leader, the teacher must also be a skilled manager. Establishing ground rules that make sense and are consistently utilized is a critical factor in making the differentiated math classroom successful. One of the most important ground rules that needs to be established is uninterrupted small group instructional time. When the teacher is working with a small group, other students need to find out the answers to their questions through means other than asking the teacher. If the classroom rules include no talking or no getting out of your seat, we’ve created contradictions in the expectations. To avoid classroom contradictions, teachers need to encourage students to engage in productive conversations which include clarification of directions and math discourse. Additionally, students need to know how to get the manipulatives they need to solve math problems. Teachers don’t have time to be waiters and waitresses, bringing things to students. Students need to do these things themselves, so teachers can focus on teaching.

Managing and leading the classroom involve setting realistic expectations and focusing on progress. One year I had a class of first graders who needed constant encouragement and support. It seemed the only way the students could accomplish a task was when I was right beside them. To make differentiated instruction a reality, I had to step back and reflect upon what was happening. I wanted the students to be independently productive, so I could work with small groups; yet I had become a crutch for nearly all of my students. I enabled them to be completely dependent upon me. I fixed this problem one step at a time. First, I provided a simple task for everyone and explained to them that I would be working with a small group without interruption for two minutes. Other students were to work without my help for the two minute period. We accomplished this small step and celebrated our success! Students shared how they solved their own problems. Jonathan asked Maria if he could borrow a pencil. Sharon asked Darrell if he wanted some base ten blocks to use. Tony asked Marc to re-read the directions to him. The students were empowered by their two minutes of success, and I used this experience as a springboard.

Next, we successfully implemented three minutes of small group time during math. Within a week, we were up to seven minutes. Within a month, I was able to work with three small groups for at least ten minutes each, allowing me to

target the instruction for all students. Now don't get me wrong, I couldn't just ignore the rest of the class for thirty minutes straight. Between groups I checked on the progress of students and provided hurdle help as needed. Many times I helped my small group get started on a task and then left them working so I could make a quick walk around the classroom to monitor progress and provide support and academic challenges to those who needed it. As I began to differentiate the independent work for students, I met with my small groups for less time until we could build the structure back up to where we had previously been. It was the modified management and leadership that opened the doors to multiple opportunities for targeted, differentiated math instruction.

## Types of Ongoing Assessment

*To know what to address, we need to assess!*

To know what to teach as we target instruction in mathematics, we need to utilize assessment. In a recent workshop, I referred to assessment as education's A-word. Some people disregard the A-word. Others are afraid of it. Still others use it without really understanding what it means. Honestly, assessment is not a bad word—just misunderstood, no wonder. We have formative and summative assessments, direct and indirect assessments, qualitative and quantitative assessments, embedded assessments, developmental assessments, and authentic assessments (just to name a few). As educators think about assessment, other connected thoughts emerge—evidence, feedback, records, grading, reporting, rigor, reliability, validity, objectivity, subjectivity, standards, blueprints, dipsticks, portfolios, quality, and fairness. So many thoughts and emotions are wrapped up in the word assessment.

While all of these types and thoughts of assessment have value and are applicable to the classroom, it is their very existence that makes things so overwhelming for teachers. Which type of assessment is best? When will I have time to teach if I spend so much time assessing? Can't it just wait until I'm finished teaching? To achieve the National Council of Teachers of Mathematics (NCTM) (2000) recommendation that "assessments should support the learning of important mathematics and furnish useful information for both teachers and students" (372), mathematics educators need to focus on using assessment in purposeful, productive ways. In the classroom, we need to use assessment to reveal the information we currently need to teach all of the students. Essentially, the assessments drive the instruction and can be categorized in three basic forms—formal, informal, and personal.

## **Formal Assessment**

Formal assessments include tests, quizzes, exams, diagnostic tools, and other traditional forms of assessment. Many formal assessments are designed to evaluate the overall effectiveness of the instructional program. While these types of assessments can be beneficial in ways that may help educators evaluate student competency and may pinpoint instructional areas that need improvement, they are not particularly useful to teachers on a daily basis. Often these tests are designed poorly because feedback on performance is not timely or inclusive and the tasks often have little value to students (Wiggins 2003). Give the test, post the scores, and the school year is over. Or give the test, record the grade, and move on to the next chapter—these cycles do not give the teacher a chance to use the assessment data to target instruction.

Fortunately, not all formal assessments have to be used in a summative manner. Teachers can actually use the information to revise teaching and learning! These are the formal assessments that directly impact classroom instruction. The chapter test can be used as a preassessment. A quiz or a diagnostic tool can be given in the middle of the lesson or in the middle of the unit. We can make some reteach plans after the postassessment. In these examples, formal assessments are used in formative ways.

Formative assessment gives feedback to teachers and students in ways that influence instruction. Even when a test is given at the end of the unit, the teacher can still use the information to create reteach groups and challenge groups that can meet during subsequent chapters of study. The reteach group focuses on concepts that preceded the current concept of study. The challenge group focuses on concepts above the current concept of study. The point is using the information from the assessments, not merely recording results in a grade book or on a report card.

## **Informal Assessment**

Most of the ongoing assessments that I use to target instruction in mathematics are informal. The goal is to find out what students know before, during, and after instruction. As recommended by the National Association for the Education of Young Children (NAEYC) and NCTM (2002), “Early childhood mathematics assessment is most useful when it aims to help young children by identifying strengths and needs so as to inform teacher planning” (12). The focus is on improving student learning through the use of purposeful data. Informal assessments such as index questions, observations, interviews, and journal reviews give teachers information they can use to guide instruction.

### **Index Questions**

An index question is a very simple way to gain knowledge of where students are in their understandings of a particular concept. A teacher poses questions or gives tasks that are directly related to what is being taught (or will be taught next). The questions or tasks give an “index” of where each student is in terms of their level of understanding. Typically the questions or tasks require more than a one-word or one-number response. Students have an opportunity to show what they know about the given topic. Each student writes or draws a response on a note card or piece of paper. The teacher collects them, sorting them by levels of understanding. Index questions may be administered to students as preassessments, assessments during instruction, or assessments after the instruction.

These formative assessments are not intended to be time consuming. Typically, the students have five minutes or less to write or draw responses and then the teacher uses a few minutes to sort the indexes. Sorting the indexes into levels of understanding becomes faster and easier for teachers as they gain more experience. Some teachers prefer to sort the indexes when the students are not in the classroom (during planning time or after/before school). Other teachers sort the indexes while the students are doing a practice problem or other independent activity.

I always encourage teachers to think about the indexes as at-a-glance knowledge indicators. What do you think the student knows about the topic? What evidence do you see? Does the index show that the student has a strong understanding, a mediocre understanding, or a weak understanding? Often the student responses fall into three groups—students who already know; students who know something about the concept, but don’t have a complete grasp of the concept; and students who are struggling with the concept. Sometimes there are only two groups, sometimes there are four groups—it varies with each class and each math topic. The teacher decides how many groups are needed and, most importantly, how many groups are manageable. Having too many groups can be a disaster for the teacher. Let’s say the indexes show eight levels. Just the thought of eight different levels of instruction is overwhelming. Not to mention the fact that the more groups, the less time the teacher can spend with each group. If there are too many levels, combine some. Try to make a quick decision about which group to put the child in based on the evidence the student provides on the index and based on what the teacher knows about the student. If it is later determined that a student should be in a different group, simply move the student—that’s the beauty of purposeful, flexible grouping of students.

Basing the quick sort of indexes on evidence is key. The last thing we want to do is assume a student should be in a specific group. We are trying to move away from the stereotypically stagnant achievement grouping. The groups

need to be formed based predominately on the evidence presented on the index. Using index questions allows the teacher to cater the instruction to the current needs of each student. Index questions are such cornerstones for targeted instruction that an entire chapter of this book is devoted to their use with number and operations in prekindergarten through second grade.

### **Observation**

Observation is another important type of informal assessment. By observing students engaged in mathematics situations, educators can assess the strengths and needs of students. It is essential that we uncover what students know before, during, and after instruction. In this way, informal assessment is dynamic and ongoing, not merely something done at the end of a unit. For example, as kindergartners count objects, the teacher can take notice of each student's mathematics knowledge. These observations may be recorded in many forms—*anecdotal records* (which include verbal responses and behaviors), *note cards*, and *checklists*. Through observation the teacher is looking for these skills and others:

Student has one-to-one correspondence.

Student knows the rote counting sequence.

Student can keep track of what has already been counted.

Student can name the number of elements in the set.

Student knows the quantity even if the objects are rearranged.

The goal in this situation is to find out where students are in their understanding of numbers and counting. With this information, educators can then target instruction for their students. In turn, assessment through observation can be incorporated to focus on the progress of learning. Strengths and potential gaps in the students' knowledge are revealed and addressed. As Copley (2000) specifies, "Teachers must observe children's actions, behaviors, and interactions and listen carefully as they talk" (24). Undoubtedly, observation is a helpful way to gather information to target differentiated instruction.

### **Interviews**

Interviews are another way to informally assess students. Asking them how they solved problems and why they used specific strategies reveals a great deal about their mathematical abilities. Manipulatives often enhance students' abilities to express their mathematics understanding. Hence it is important to provide an assortment of manipulatives for students to use to show, explain, and prove their thinking. For example, by

inviting first grade students to show the number 28 in a variety of ways, the teacher can pose questions to determine the students' understanding of how numbers are composed and decomposed. Insight into student understanding may be revealed through interviews that address these and other questions:

Can the student show and explain 28 as 2 tens and 8 ones?

Does the student understand that 28 can also be represented as 1 ten and 18 ones?

Is the student comfortable composing (building) and decomposing (breaking apart) numbers?

How developed is the student's sense of how this number is related to other numbers?

As students explain and justify solutions to math problems, educators learn which concepts students know and which concepts students need to learn. Assessments that inform practice enable mathematics educators to meet students where they are as they target the math instruction.

### **Reviewing Journals**

Even very young students can and should keep math journals. Math journals provide students with opportunities to use pictures and words to characterize and describe mathematics situations. Math journals serve as a powerful way for students to communicate and represent their thinking. Reviewing journals provides teachers with additional means of informal assessment. This practice enables teachers to examine evidence of students' growth in mathematics understanding over time. While reviewing math journals, the teacher is looking for answers to these and other questions:

Can the student accurately apply mathematical concepts?

Does the student use math vocabulary?

Which strategies are evident?

Is the math thinking accurately represented?

As with index questions, observations, and interviews, journals can be used to assess math knowledge of students before, during, and after instruction. Clearly, the best assessments are those that align with productive instructional strategies. Allowing students to "show" what they know in the same ways they "learn" new knowledge is a powerful way for the teacher to target instruction.

## **Personal Assessment**

Personal assessment is unlike other forms of assessment in one very distinct way. The teacher is not the evaluator. Students appraise their own levels of math knowledge. At first, the thought of students deciding what and how much they know may seem a little daunting! As teachers, we feel so accountable and spend a great deal of time evaluating. So it is sometimes difficult for us to break away from being the judge of a student's knowledge or comfort level; however, it is empowering to students when we put them in charge of assessing these things.

Okay, I know the questions that follow. And yes, I have worked with many students who think they know more than they actually do. Likewise, I have worked with many students who are not as confident as they could be about their knowledge. And, yes, some students initially choose to mimic their friends just to get to work with them. Still other students are embarrassed if they do not know something. These are some of the reasons teachers may shy away from personal assessments. They worry students' assessments of their knowledge are not accurate. They worry students will say they know when they really don't or that students will think there is a smart group and a stupid group. As teachers, our job is to teach the students how to accurately assess their levels of understanding within a safe classroom community. Students need to know that no one (including the teacher) will make them feel inadequate because they do not know something. It is all about knowing what you know and learning what you need to learn next. It takes time. With some students, it takes more time. But the time is worth it because if students can accurately judge where their understanding lies, they are empowered to move on in their learning journeys, which will ultimately save your time.

One of the ways to help students gain accuracy in assessing their level of knowledge is to combine personal assessments with index questions. Some of the indexes can include a scale. Students are asked to rate their levels of understanding when they respond to the index question. Combining indexes and personal assessments helps the teacher know how well the students judge their knowledge of any given math topic.

Options for different types of personal assessments are numerous. The more you use them, the more various formats come to mind. I started with the common thumbs-up signal. Thumbs-up if you understand the concept. Thumbs-sideways if you understand some of the concept. Thumbs-down if you are confused about the concept. Like many teachers, I have used these signals to gather a general sense of where my class is functioning at any given time. In the past, I asked for the thumb signals and then continued with whatever I was doing. Now I use these signals as a way for students to personally assess where they are in their current learning journey. I respond to the thumb

signals. Perhaps I need to meet with my thumbs-up group to give them a challenge. Or perhaps I need to meet with my thumbs-down group to provide some support. The point is the teacher needs to use the information to make adjustments in the level of cognitive demand required for the task and to target instruction for specific groups of students.

Students can evaluate their levels of understanding on different types of scales. Using vertical and horizontal number lines, students rate personal degrees of understanding. Students can use individual number lines as scales to mark levels of understanding, as shown in Figure 1-1.

The scales can have any given range. With primary students, I usually begin with simple whole-number scales such as zero to ten. Later, I increase the complexity of the scale by adjusting the range. Hearing a student announce, “I understand this idea at a level of about three-fourths,” is music to the math teacher’s ear! Students are self-evaluating and using mathematics language.

Using class line plots can also be beneficial. Students place a name sticker in a location on a line that indicates knowledge level of or comfort level with a specific concept (see Figure 1-2).

The line plot is then used to group students purposefully and to target instruction using the personal assessment information.

Graphs and comparison circles are additional ways to gather personal assessment information. Students place name cards in specific columns, rows, or locations to indicate their level of understanding. As shown in Figure 1-3, the bar graph labels offer ways to indicate degrees of understanding of or comfort with the math topic.

If the class is learning about composing and decomposing numbers, the comparison circle labels could be “I know how to make twelve with groups of two” and “I know how to make twelve with groups of four.” At a glance the teacher can see which students need more support and which students need more challenge (see Figure 1-4).

Personal assessments can also be used before, during, and after instruction. Most of the time students gain knowledge from the instruction.

**FIGURE 1-1**

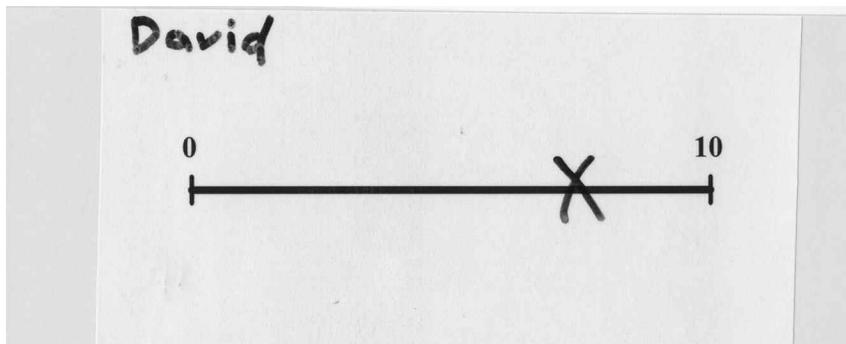


FIGURE 1-2

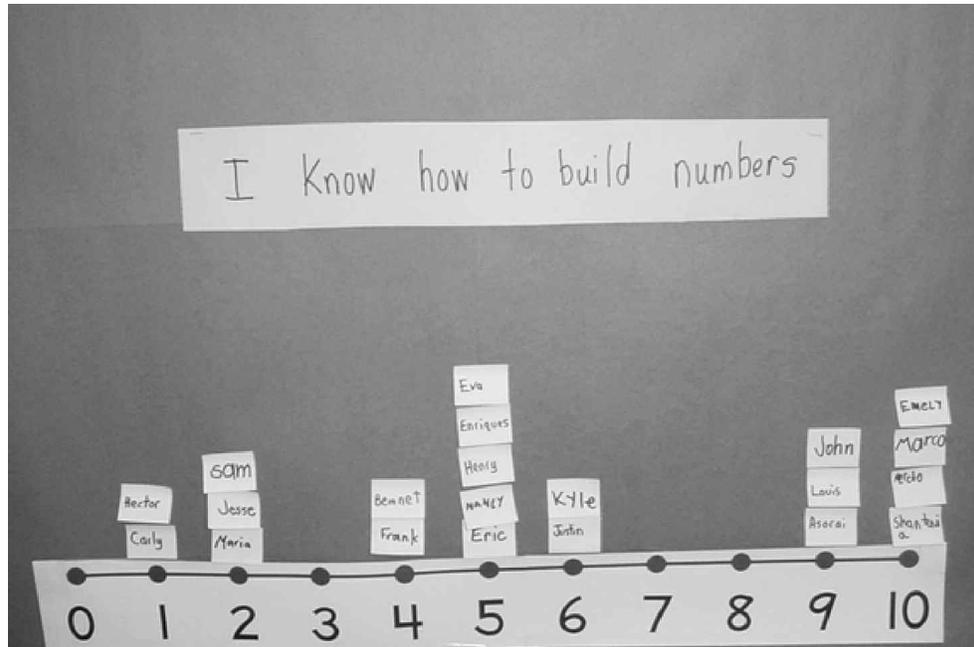


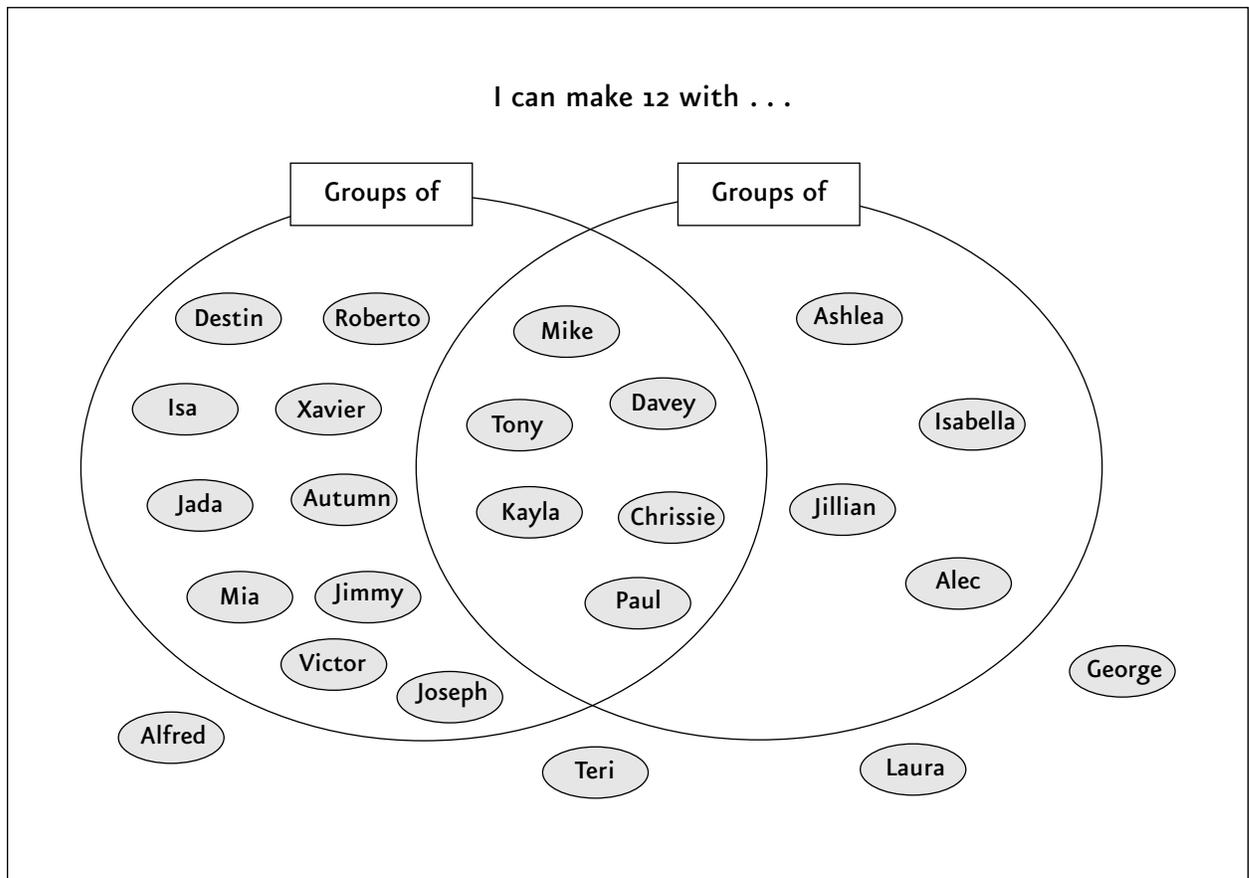
FIGURE 1-3

How much I know about

**Equal**

A lot	Francis, Liza, Aisha, Tyler, Luis, Paige
Some	DeeAnn, Sonny, Mandy, Perry, Matt, Rodie, Gene, Patrick
Little	Lara, Yvette, Hector, Megan

**FIGURE 1-4**



Because we use stickers or magnetic name tags, students can easily relocate names or use before and after name tags to show the potential increase in knowledge or comfort level. Student comments such as, “I’m going to move my name up on the line because I get it now!” prove the power of personal assessment tools. By encouraging students to accurately determine where they are in their learning, the teacher is afforded a quick and easy way to form learning groups. The benefits multiply because the graphic organizers used to determine learning levels actually open the door to more math thinking and more math discourse! Using graphic organizers to gain personal assessment data is such a new and exciting idea that an entire chapter of this book (Chapter 7) is devoted to providing examples and explanations—specifically for prekindergarten, kindergarten, first grade, and second grade.

## The Differentiated Instruction in Mathematics Cycle

*The phases of differentiated math instruction in action!*

There is a cycle of actions that illustrates a powerful way to implement differentiated math instruction.

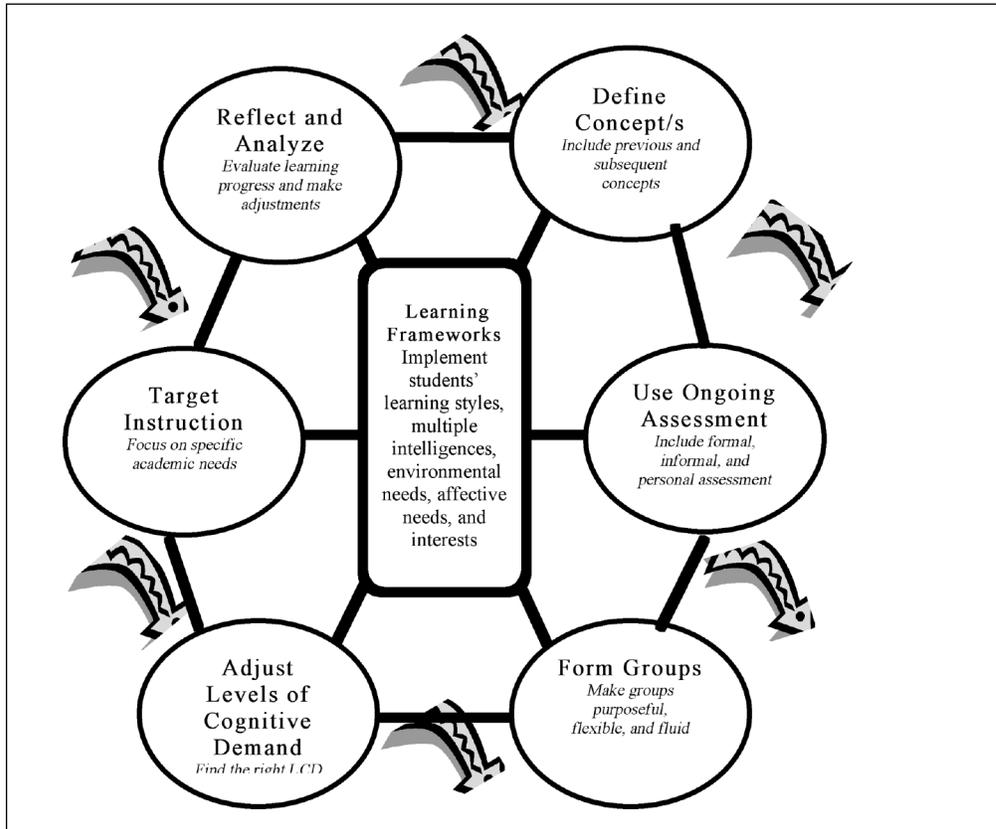
The steps include (see Figure 1–5);

1. Define Concepts: Include previous and subsequent concepts
2. Use Ongoing Assessment: Include formal, informal, and personal assessment
3. Form Groups: Make groups purposeful, flexible, and fluid
4. Target Instruction: Focus on specific academic needs
5. Adjust Levels of Cognitive Demand: Find the right LCD
6. Reflect and Analyze: Evaluate learning progress and make adjustments.

The process begins with defining the concepts that need to be taught. The teacher identifies the on-grade-level benchmarks or objectives and the connected previous and subsequent concepts. This process gives the teacher content knowledge for accurately and efficiently differentiating mathematics instruction. The next phase is the use of ongoing assessment. The teacher uses formal, informal, and personal assessment methods to gather data about where students are in their levels of understanding of the math concepts. The teacher has many choices. Perhaps he decides to give the students an index question to assess knowledge. When the data are collected, he uses the information to form purposeful, flexible, and fluid groups. The groups are not arbitrarily or randomly assigned. The teacher may group students homogeneously or heterogeneously depending on the goals of the teacher and the needs of the students (more about purposeful, flexible, and fluid group structures will be addressed in Chapter 3).

After forming the groups, the teacher adjusts the levels of cognitive demand for students. Some tasks may need to be simplified. Other tasks may need more complexity. This adjustment in the level of cognitive demand may be present in the assignments students work on, the questions that are posed to them, and the follow-up tasks they are given. Sometimes the targeted instruc-

FIGURE 1-5



tion occurs simultaneously with the adjustment of the levels of cognitive demand; other times the targeted instruction follows or precedes it. In any case, the teacher focuses on the explicit academic needs of the students. If some students have a gap in understanding a specific concept, the teacher may have a small group mini-lesson on the topic which can serve as the targeted instruction. If other students already understand the on-grade-level concept, the teacher may meet with this group of students to provide challenge by targeting instruction to the next grade level concept. The options are many, but the goal is singular—focus on the needs of the students.

The differentiated instruction cycle encourages the teacher and students to analyze and reflect. How is the learning progressing? What changes need to be made to set students up for success? Certainly, reflection and analysis can and should occur at all times during the cycle; we don't have to wait until the end of the lesson or school day. However, we need to make sure we include reflection and analysis in our educational practices.

Incorporating students' learning frameworks serves as the backdrop of the differentiated instruction in mathematics cycle. Learning styles, multiple

intelligences, environmental preferences, affective needs, and interests are ways to connect students with positive learning experiences. Students' motivation and commitment are increased as teachers incorporate learning frameworks. Each of the phases in the differentiated instruction in mathematics cycle can occur at anytime, and they do not have to happen in a specific order. The differentiated instruction in mathematics cycle is intended to provide a foundation for teachers who want a map of a way to successfully implement differentiated mathematics instruction. The cycle also gives us insight into the most important ideas to consider when differentiating mathematics instruction.



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