

# **The Differentiated Math Classroom**

*A Guide for Teachers, K–8*

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Figure 8.3: Gregorc Mindstyle™ Profiles for Miki Murray and Jenny Jorgensen from the Gregorc Style Delineator™. Used by permission of Anthony Gregorc.

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## Chapter 4

# A Problem-Solving Platform

*Problem solving should be the central focus of the mathematics curriculum. As such, it is a primary goal of all mathematics instruction and an integral part of all mathematical activity. Problem solving is not a distinct topic but a process that should permeate the entire program and provide the context in which concepts and skills can be learned.*

National Council of Teachers of Mathematics,  
*Curriculum and Evaluation Standards for School Mathematics*

### Problem Solving

In 1997 I went back to the classroom to experience using the problem-based curriculum, *Connected Mathematics*, in a multigrade 7–8 program. My biggest worry was two grade levels in the same class period. I had experienced twenty-five years of challenging and often frustrating work with one grade level at a time. What really worked for the seventh- and eighth-grade combined group were the engaging problems that allowed students to differentiate for readiness—the biggest issue—for themselves! Every problem was accessible and the students took it from there.

When students are engaged in the exploration and solution of a problem that “grabs them,” they will go as far as they can, or as far as their readiness takes them. Some will attain only part of the solution and some will go well beyond and make important connections and hypotheses for themselves. Those who might be considered “below grade level” are not shut out of the closing discussion, which is an additional opportunity for conceptual development while considering the thinking of contributing classmates.

I worked hard to facilitate an environment to support and maximize their learning. I was very much the reflective/responsive teacher. I tweaked and planned for every available moment and each student need, but the key was the *problem-solving platform*. How powerful that was and is!

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A problem-solving platform is a mathematics curriculum that consistently draws students into mathematical inquiry through stories, situations, or scenarios that challenge students with intriguing problems. The problems are embedded with rich content waiting to be uncovered. The revelation of the mathematics takes place in an atmosphere of shared strategies at an individually determined pace. By that we mean that each child grapples with the mathematics at her or his readiness level while operating in a collaborative environment where support comes from the perspectives of other students as well as the teacher.

## **Good Problems**

Good problems are essential to this process. They are irresistible and open, come in a variety of forms including games and puzzles, can emerge from a teachable moment, and invite persistence. The contexts vary and shift with the needs and interests of the children. Good problems also allow the teacher access to student thinking as the work (exploration) progresses.

Here are two of my favorite problems to begin the year.

1. I hold up a classroom chessboard and ask, “How many squares do you see?” Hands are raised. Wait time is honored. Suz is called on and responds with “sixty-four.” Asked how she knows it is sixty four, she explains that there are eight squares on each edge so she multiplied  $8 \times 8$ . There is a pause before several tentative hands are raised. Jonathan sees the single large square outline of the chessboard. Kristin then eagerly comes to the chessboard to outline a  $2 \times 2$  square with her finger. Soon the issue of all the different size squares is raised and the exploration begins, individually or with a partner. Figure 4.1 shows Nick’s explanation for his solution.

The chessboard problem differentiates itself because it is open to different processes for solving. All students have access; they need only understand squares and be able to count. The idea of different-size squares can be scaffolded if necessary. Students can use different strategies, such as make a simpler version, look for patterns, sketch with different colors on grid paper, and so on. Because students are required to write about their solution processes, there are multiple opportunities to observe student understanding—during the exploration and by reading the students’ explanations. A great deal of mathematics is explored during both the solution process and the summary of shared solutions and discoveries.

2. *Petals Around the Rose* is a casual challenge that intrigues students. I read about it in an article written by Marie Appleby, a middle school math teacher from South Hadley, Massachusetts (Appleby 1999). The game begins by rolling five dice while circling the room and chanting: “The name of the game is *Petals Around the Rose*. The name is very important. For each roll of the dice, there is one answer, and I will tell you the answer.” Continue

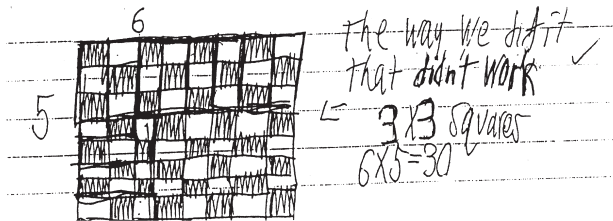
## Math Write Up

We did a problem called Chessboard where the objective was to find the total number of squares in an eight by eight (sixty-four square) chessboard.

You have to count every possible square, meaning any size: 1x1, 2x2, 3x3, 4x4, 5x5, 6x6, 7x7, and 8x8. Then you add up all the numbers of all the different squares and you have the answer or the sum. The squares can also overlap.

When we started, we just tried the obvious thing – we started counting. Soon it became clear that that wouldn't work, so we started looking for a formula or a pattern. We thought we had found the pattern when we multiplied the number across by the number down. It worked for 2x2 but nothing else.

We tried various ways of solving it using multiplication, because we knew the formula would have something to do with multiplication because it was a square, so all four sides are equal. We thought that would make for an easy solution involving multiplication. [insert diagram]



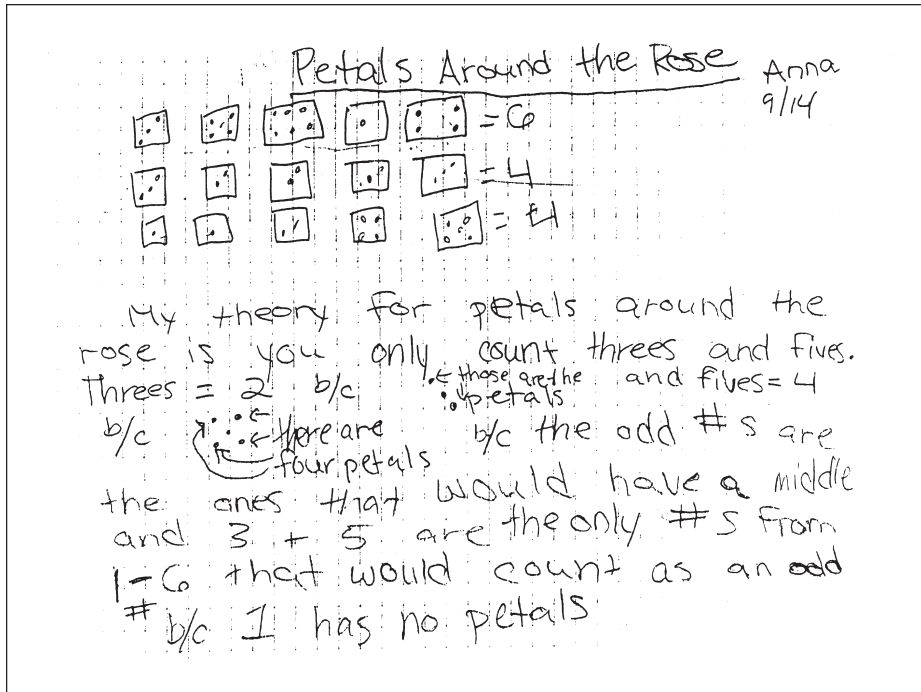
We finally figured out what we did wrong and how we should have done it. The solution did involve multiplication though.

The way it works is for each size of square (e.g. 2x2, 3x3, 4x4, etc.) you count how many can fit along one side, then square that number because a square has four equal sides. Then you add all the numbers up and you've got your answer. For a bigger square you'd just do the same thing, count how many can fit on one side and square it. [insert table with solution]

Size of square	# of squares on each side	Equation	Total # of squares
1x1	8	$8^2$	64
2x2	7	$7^2$	49
3x3	6	$6^2$	36
4x4	5	$5^2$	25
5x5	4	$4^2$	16
6x6	3	$3^2$	9
7x7	2	$2^2$	4
8x8	1	$1^2$	1
			204

**FIGURE 4.1**—Nick's problem-solving.

rolling and give only answers (for example one, one, three, four, and six have an answer of "two"), presenting the students with data without questions or conditions. The students are drawn in, trying to figure out a connection between the dice they are seeing and the answers being given, as you should be. See Anna's conjecture in Figure 4.2 for an explanation of how answers are determined.



**FIGURE 4.2**—Anna's solution for, *Petals Around the Rose*.

The stage is set for students to begin verifying their ideas without teacher input. Give no feedback. They now “need to know.” Continue the game over several days in short bursts as a sponge activity with intermittent group processing and data collecting prompted by questions such as “What do we know about *Petals Around the Rose*?” Ask students to write their thoughts about the game in their journals. Establish the rule that the only way a conjecture is considered is with a roll of the five dice and a correct response from the conjecturing student. Swear students to secrecy once their solution is confirmed. Institute this constraint early so that all students have the opportunity to eventually discover the pattern for themselves. Once a student is able to crack the code, that is “see the rose and its petals,” that student can run the game for others.

Glenn, a fifth- and sixth-grade teacher, used the game this year and enhanced its benefits by establishing the knights of the order of the Rose. A ceremony knighted each solver who was then certified to run the game for classmates.

This casual informal game for all ages makes several statements about problem solving and differentiation. First of all, it’s fun. It is challenging and engaging at various levels for all students. They can use a number of strategies such as guess and check, list known facts, draw diagrams and samples, write about it in a journal, and make the problem simpler by rolling fewer dice.

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Such experiences led us to use engaging problems embedded with worthwhile mathematical tasks as the centerpiece or platform for differentiating instruction in mathematics. For three years I watched *all* of my students grow in their mathematical knowledge and understanding (some exponentially), from the most challenged or disengaged to the most gifted—all in the context of problem solving. I was amazed to realize, after twenty-five years of grouping with basal texts, individualized programs, creating units and math labs, using cooperative learning and homogeneous grouping, that the problem-based curriculum was the format that empowered me most as a teacher of mathematics and empowered students the most as learners of mathematics.

So how does this work for an overarching plan? Start the study of each major concept or fundamental math idea with a juicy problem in which the content is embedded. For example, in the February 2006 issue of *Mathematics Teaching in the Middle School*, a group of middle school teachers share the problem they use to introduce their algebra units for grades 5–8 that have the following range of conceptual goals: to represent real-world situations with tables and graphs, to introduce linear equations, and to bridge linear equations and systems of equations.

If you have \$10 to spend on \$2 Hershey's bars and \$1 Tootsie Rolls, how many ways can you spend all your money without receiving change? All chocolate, no change! (Hyde et al. 2006, 262–63)

Although simple, the level of difficulty is easily increased by changing the dollar amounts. The task is simplified for younger or special needs students by giving them play money. The teachers spend from two to five class periods uncovering the rich connections and unlimited extensions. Students are absolutely engaged when the problem is introduced in the presence of a giant chocolate bar and a basket of Tootsie Rolls!

### **Criteria for a Good Problem**

Teachers are constantly challenged to select or create suitable problems even if they work with problem-based curricula. They sometimes want or need to adapt a lesson to match student interests, readiness, or sensitive issues.

For example, in the February 2006 issue of *Teaching Children Mathematics*, Christina Nugent, a fifth-grade teacher in Dubuque, Iowa, describes a problem she created for her at-risk school classroom. She posed the question, "How many blades of grass do you think are on a football field?" and a powerful differentiated unit of work ensued that included number sense, estimation, measurement, area, computation practice, and communication (writing is used in all aspects of her mathematics class). The work also foreshadows and prepares for more in-depth development of proportional reasoning. She calls this a high-quality problem for many reasons. Football is of high interest, the context is meaningful and real world, it piques students' curiosity, and there is no obvious way to solve the problem (Nugent 2006).

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The following criteria can be used as guidelines for developing or evaluating good problems. The list represents the thinking of several major curriculum researchers and developers. A problem does not need to address all of the issues but should fall within these guidelines to meet standards of worthiness, what Christina Nugent refers to as “high quality.”

- Solving the problem leads to significant mathematics.
- The process leads to (foreshadows) important mathematical ideas for future work.
- The problem is open-ended; it can be approached in multiple ways using various strategies.
- There are different ways into the problem: it is accessible to students with different strengths, needs, and experience.
- The problem is interesting and engaging to a wide range of students.
- The problem has various solutions that encourage justification or comparison.
- The problem leads to higher-level thinking and discourse.
- The constraints provide direction without limiting thinking and exploration.
- The problem strengthens conceptual development related to important mathematical ideas.
- The problem provides the opportunity to practice key skills.
- The problem allows teachers to assess how and what students are learning and to identify needs. (Mokros, Russell, and Economopoulos 1995; Lappan and Phillips 1998)

The Chessboard, Petals Around the Rose, and All Chocolate, No Change problems fall within the guidelines for high-quality problems. In addition, they seamlessly lend themselves to differentiating for readiness and learning styles. Here are more examples.

1. Basic multiplication facts are used constantly in our math work. How many *different* products result from multiplying the whole numbers one through nine, two at a time including factors that are the same such as  $2 \times 2$ ? Record your thinking. (Posed before introducing the Product Game, a game used for practicing basic facts that has a matrix of products as a game board with the factors below. The products are exactly those needed for all possible combinations of any two of the factors—none are repeated as they would be in a multiplication table.)
2. How many ways can you stack six boxes? Which requires the least floor space? (Ask your own questions and create different constraints.)
3. The dimensions of a typical cereal box are about two inches wide by six inches long by twelve inches high. The shape and size of cereal boxes haven't changed much over the years. Each cereal box creates waste to recycle. Find a way to help the environment by designing a cereal box with the same volume that uses less material for the package.

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4. Jenna was visiting her cousin and the weather turned really cold. Aunt Anne went to the closet and returned with a jacket that was now too small for her cousin and told Jenna she could keep the jacket and anything she found in the pockets. She reached into her pocket and found sixty-seven cents. What possible coins might have been in the pocket?
5. Use four 4's to generate equations that equal 0, 1, 2, . . . . For example:  
 $(4 + 4) \times (4 - 4) = 0$        $4(4 - \sqrt{4}) + 4 = 12$

A good problem can emerge in various formats. It may be a question; it may be a situation; it may be the challenge in a game. Students might formulate a problem as the result of a puzzling context, or a problem might seemingly pop up out of nowhere—a teachable moment. The criteria for high-quality problems listed above give guidance for the design and selection of appropriate problems for accomplishing your curricular goals. In the following scenario, Glenn uses a carefully designed problem to help all his students develop a deep understanding of a key mathematical idea, the mean of a set of data.

### Scenario

The fifth-grade students in Glenn's class measured and recorded their heights during a unit on data. The heights were recorded on a class chart. Each student also recorded his or her name and height on a  $3 \times 5$  index card along with other personal data to be used in the unit. The day they began the study of the mean as one kind of average, Glenn took out the class data deck of cards, shuffled them, and drew five cards from the deck at random. Without revealing the names he had drawn, Glenn performed a mysterious calculation on a hidden pad of paper and announced that he had the mean height of the five students whose cards he had drawn. The challenge for the class was to identify five students in the class who have that mean height. (Not necessarily the ones drawn!)

First, Glenn explored what students thought the *mean* meant! A sound understanding of the mathematical concept of mean was critical to their investigation, so Glenn asked for volunteers for a demonstration. He selected students of obviously different heights and cut strips of gridded chart paper (with convenient one-inch markings) to match their heights. These were taped to the chalkboard, and with the students seated in their math circle, he instructed them to talk to each other about how the three paper strips representing the student heights could be made the same length without losing *any portion* of the paper strips. Using student suggestions and discussing options, a strategy was devised. The students collaboratively advised, cut, and taped the strips until they were exactly the same length. (They giggled a little about why it's advisable to use paper strips instead of the actual children for the demonstration.) In this way, the class was able to experience what it means to "even out" the data in order to determine the mean height for the three students.

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Then it was time to tackle the problem. All the students set to work with their partners or groups of four, armed with the class table of heights and whatever tools they considered necessary. They were directed to find at least one set of five classmates (themselves included) who could be evened out to the given mean height. They were instructed to record their work and prepare to justify their choice of five students and prove that the mean for those students matched the given mean. Glenn circulated among the groups, listened to and noted how different groups approached the problem, supported struggling groups, and planned for the lesson summary, next steps, and extensions. There would be something fun and interesting for any group that was able to identify the exact group associated with the given mean—to be revealed after all possible sets had been proposed and justified.

In this session, Glenn differentiated by scaffolding the concrete meaning behind *mean* and making the problem accessible to all students. He also differentiated by allowing students to attack the problem their own way, choosing whether to work with partners or a group of four, and inviting multiple solutions. Glenn’s problem was open-ended, interesting, engaging, accessible to all students, and packed with mathematical concepts and processes. It was about the students (always seductive to kids), could be approached systematically or creatively, and had the possibility for multiple solutions. It clearly matched the criteria for a good problem and was an especially good match for the kinesthetic learning style described in Chapter 3.

## **Timing**

It is important to understand that attaching an expected time to any particular problem belies the variable nature of how that problem might serve the differentiation process. The timing depends not so much on the problem as on the students, the time available, and how the teacher is using the problem—is it a daily warm-up or an introduction or the substance of a unit? What is the time available to the class? Some problems are presented at the beginning of a unit only to pique interest and won’t be tackled by the class until somewhere midway through a four-week series of investigations. This is true of all of the *Connected Mathematics* units. Also, a problem that takes one day for one student might take a week for another, as is the intended case for *Petals Around the Rose*.

## **Problem-Posing Support**

Sullivan and Lilburn in *Good Questions for Math Teaching* (2002) offer wonderfully simple three-step processes for creating a good question for any math area. One process involves working backward from a closed question and the other adapts a standard “what” question to a broader context.

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- Step 1: Identify a topic.  
Step 2: Think of a closed question or a standard question.  
Step 3: Open the closed question by including the answer and working backward to situations that might elicit such an answer as in the above scenario.

Or adapt a standard question:

Example 1. “What is a rectangle?” to “What can you tell me about this rectangle?”

Example 2. A completed addition or subtraction problem with digits from both addends and the sum replaced with question marks:

$$\begin{array}{r} 2?9 \\ + ??8 \\ \hline 67? \end{array}$$

Example 3. “What is the volume of a 2 inch by 3 inch by 4 inch box?” to “The capacity of a box of caramels needs to be twenty four cubic inches. What are possible dimensions for such a box?”

Example 4. “What is the median of the set of data {2, 10, 15, 5, 7, 9}?” to “A set of data has six scores and a median of eight. Three of the scores are two, nine, and ten. What might the other three scores be?”

Example 5. “Draw pictures of the first five square numbers” to “How does the area of a square change as its side grows?”

Example 6. The answer to the given fraction multiplication is  $2\frac{3}{4}$ . A good question is: The product of two numbers is  $2\frac{3}{4}$ . What might the two numbers be? (Sullivan and Lilburn 2002, 7–9)

Try several of these for yourself. You’ll be amazed—it really works!

When you pose such problems with differentiating in mind, the open-endedness accommodates for readiness and indeed informs the teacher of student levels of understanding. In some cases, this may indicate minilessons with various groups emerging. These problems also invite students to use their own styles in the solving process and to pose similar problems of their own to challenge others. Along with multiple solutions, products can be varied as well: some students will explain their work effectively in narrative; others need to draw pictures or want to demonstrate how they know they have a solution. In all these ways, differentiation is embedded in the problems and only has to be enhanced by how the teacher directs the work.

## Summary

In this chapter, we looked at differentiating mathematics through a problem-solving lens in order to capture the essence of the critical role that

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problem solving plays. Problem solving offers a sturdy platform for striking out and trying your hand at differentiating a mathematics program. Make use of Sullivan and Lilburn's suggestions for turning any simple exercise into an engaging problem or different levels of problems to suit your classroom needs. Within each part of the problem-solving process, there are opportunities for differentiation. Keep in mind that learning happens most effectively with moderate challenge. Adaptations and scaffolding can occur at the beginning when the problem is posed, in the middle while the exploration is taking place and you are working with individuals or groups, and at the end when the class is summarizing the work and you are assessing their understanding.



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