Our Teaching and Learning Philosophy

Depth versus Breadth

Lester Rubenfeld, a mathematician, once said, "When I'm working on a problem, I like examining it in many ways, going deeper and deeper to really understand the structure of it. It's like playing in the playground" (Kellison, Fosnot, and Dolk 2004). Our work is driven by the desire to transform classrooms into communities of mathematicians: places where children explore interesting problems and craft solutions, justifications, and proofs of their own making. Many materials currently on the market do not allow children to examine problems deeply. Breadth is the goal, not depth. We believe, however, that covering fewer topics in more depth will better prepare children for the tests they take and for higher-level courses. And the research supports our belief. In countries where curricula focus on fewer topics each year, students actually score higher on international assessments (Stigler and Hiebert 1999). As we developed these units we pushed for depth, not breadth. We wanted children to have opportunities to "play in the playground" of the problem, rather than merely complete quantities of similar problems or meaningless activities for practice.

The Landscape of Learning

The rich, open investigations we've developed allow children to engage in mathematizing in a variety of ways. We honor children's initial attempts at mathematizing, at the same time supporting and challenging children to ensure that important big ideas and strategies are being developed progressively. Our approach should not be confused with what is commonly called "developmentally appropriate practice," where teachers assess every child, ascribe a stage, and then match tasks to each child. Our approach emphasizes emergence. Learning, real learning, is messy; it is not linear. We conceive of learning as a developmental journey along a landscape of learning. This landscape is composed of landmarks in three domains: strategies, big ideas, and models.

Strategies, Big Ideas, and Models

Strategies can be observed. They are the organizational schemes children use to solve a problem; for example, they might use repeated addition, skip-count, or double and halve when multiplying, or use partial quotients or simplify first when dividing.

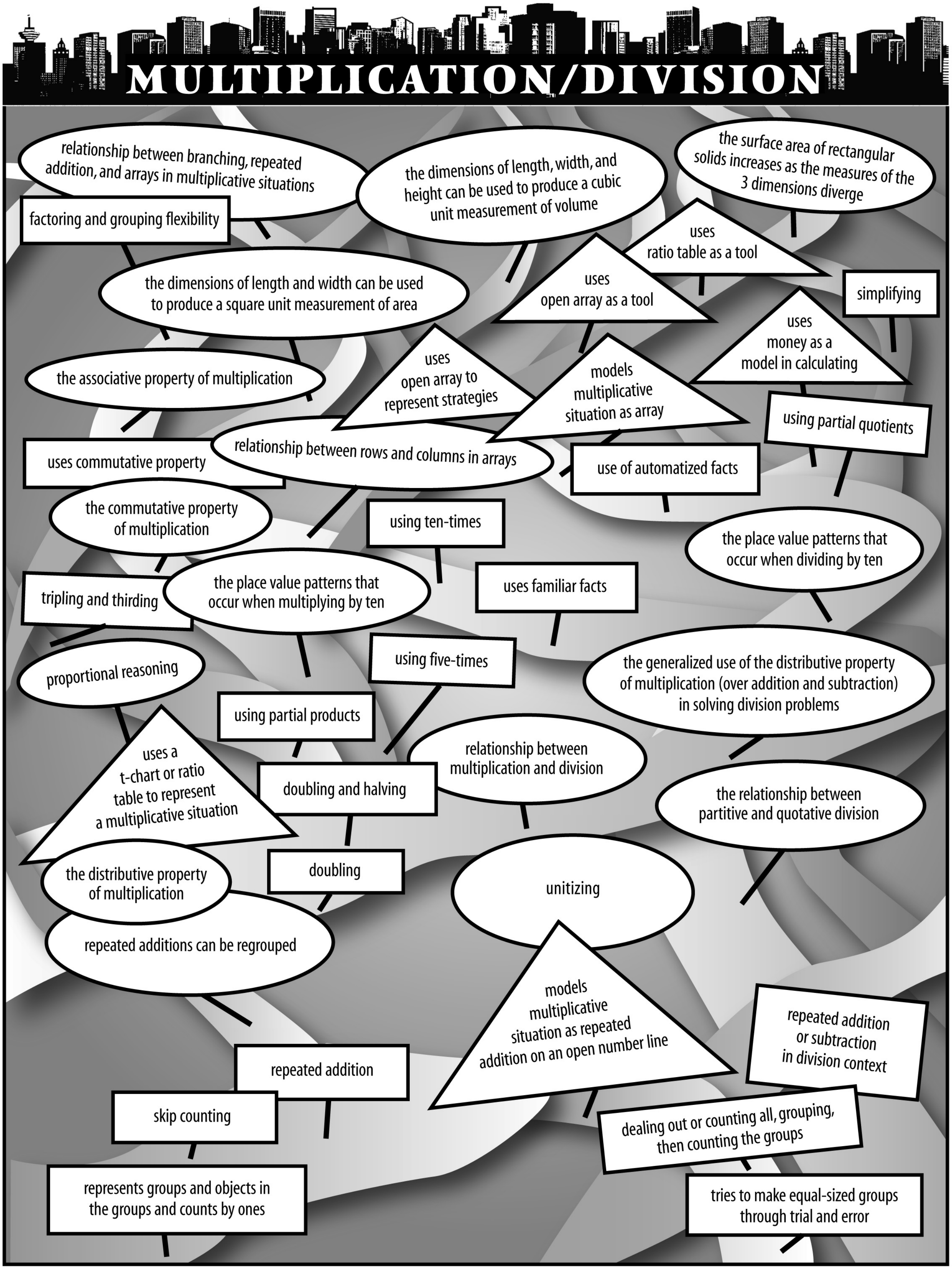
Underlying these strategies are *big ideas.* Big ideas are "the central, organizing ideas of mathematics—principles that define mathematical order" (Schifter and Fosnot 1993, 35). Big ideas are deeply connected to the structures of mathematics. They are also characteristic of shifts in learners' reasoning—shifts in perspective, in logic, in the mathematical relationships they set up. As such, they are connected to part-whole relations—to the structure of thought in general (Piaget 1977). In fact, that is why they are connected to the structures of mathematics. Through the centuries and across cultures as mathematical big ideas developed, the advances were often characterized by paradigmatic shifts in reasoning. That is because these structural shifts in thought characterize the learning process in general. Thus, these ideas are "big" because they are critical ideas in mathematics itself and because they are big leaps in the development of the structure of children's reasoning. Here are some of the big ideas you will see children constructing as they work with the materials in this package: unitizing, multiplication is distributive (the distributive property), and factors can be grouped in a variety of ways and orders (associative and commutative properties).

Finally, mathematizing demands the development of *mathematical models.* In order to mathematize, children must learn to see, organize, and interpret the world through and with mathematical models. This modeling often begins simply as representations of situations, or problems, by learners. For example, learners may initially represent a situation with connecting cubes or a drawing. These models of situations eventually become generalized as learners explore connections across contexts—for example, using graph paper arrays as blueprints in designing new boxes for Muffles' assortments, determining the number of truffles in various boxes when only the dimensions are known. Generalizing across contexts allows learners to develop more encompassing mental models to think about situations with—for example, the blueprint becomes an open array to explore the use of partial products. At this point, teachers use the emerging model didactically, representing children's invented computation strategies for multiplication and division on an open array. This stage bridges learning from informal solutions specific to a context toward more formal, generalizable solutions—from models *of* thinking, to models *for* thinking (Beishuizen, Gravemeijer, and van Lieshout 1997; Gravemeijer 1999). Models that are developed well can become powerful tools for thinking.

Learning as Development

Now picture a landscape. The landscape for multiplication and division is on page 16. On the horizon is a deep understanding of these topics. Along the way are many developmental landmarks—strategies, big ideas, and ways of modeling that as a teacher you will want to notice, support the development of, challenge learners to construct, and celebrate. The units in this package are designed to support children on this journey. Each unit has a different focus and zooms in on a section of the landscape. You will find this information on the first page of the overview in each unit.

Teaching mathematics is about facilitating mathematical development. *This means that you cannot get all learners to the same landmarks at the same time, in the same way, any more than you can get all toddlers to walk at the same time, in the same way!* All you can do is provide a rich environment, turn your classroom into a mathematical community, and support the development of each child in the journey toward the horizon.



The landscape of learning: multiplication and division on the horizon showing landmark strategies (rectangles), big ideas (ovals), and models (triangles).